Relativity

Special Theory of Relativity

Michelson-morley Experiment :

Its significance : Light is a form of transverse wave motion. It can pass through material medium like glass, water etc. also through vacuum. But wave motion always requires a material medium to travel. In order to put the wave theory of light on a sound footing it was supposed that a hypothetical medium which was given the name 'ether' exist everywhere, even in vacuum.

The velocity of wave motion is given by $v = \sqrt{\frac{E}{\rho}}$

Where E is the elasticity of the medium and ρ the density. The velocity of light has a large value of 3×10^{10} cm/sec. Hence 'ether' must have large elasticity & very low density. These two are opposed to each other and therefore, there were grave doubts regarding the actual existence of 'ether'.

To prove the existence of 'ether' attempt was made to find the velocity of drift between the ether and the earth. If ether is assumed to be at rest the velocity of the earth relative to ether will be its orbital velocity which is about 18.5 m/sec and the velocity of the ether drift relative to the earth will be equal in magnitude but opposite in direction.

To detect the presumed motion of the earth through ether, Michelson devised an ingeneous experiment with the help of Morley using his interferometer, the experiment is classically known as Michelson Morley experiment.

In Michelson's interferometer a beam of monochromatic light is split into two parts by a half silvered glass plate G. A part is transmitted through it and reflected at the front silvered mirror M_2. It retraces its path and after reflection from the back surface of G enters the telescope T.

The other part after reflection from the silvered back surface of G, falls on the mirror M_1, retraces its path and after refraction through G enters the telescope T. The two beams therefore, produce interference if the optical path $AB = AC = d$.

If the velocity of the apparatus (or earth) relative to the ether is ϑ from left to right in the direction AB, then the velocity of ether drift is also ϑ from right to left.

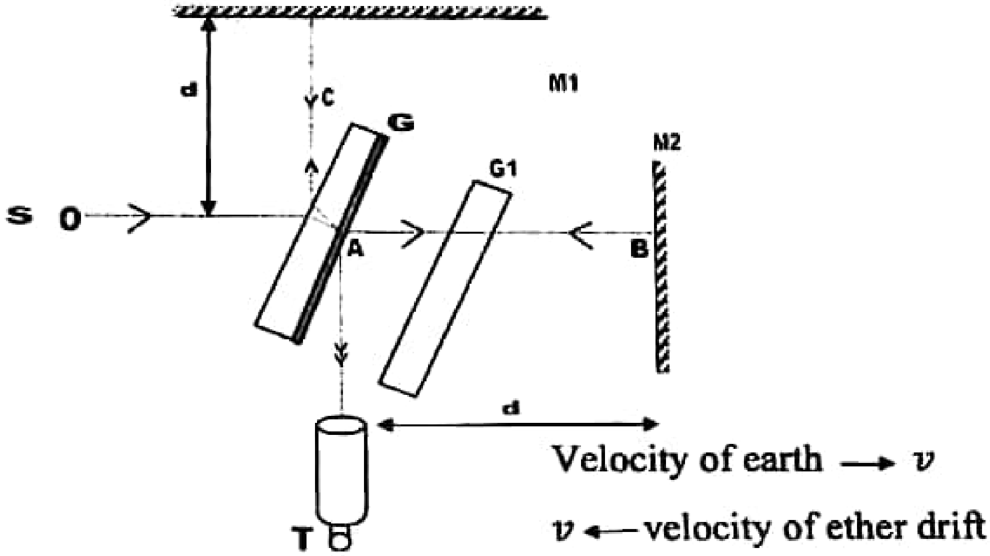


Fig 1

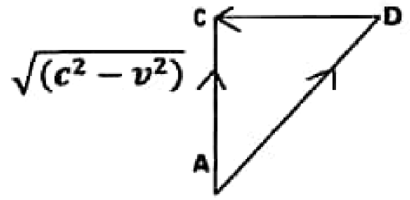


Fig 2

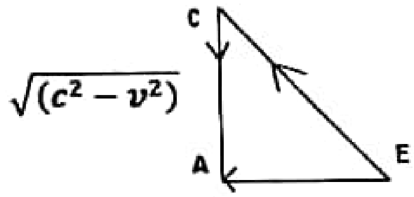


Fig 3

If C is the velocity of light then time taken by light to go

from A to B $= \frac{d}{c-v}$

And time taken by light to go from B to A

$$= \frac{d}{c+v}$$

$\therefore$ Total time taken by light to go from A to B and back to A

$$T_x = \frac{d}{c-v} + \frac{d}{c+v} = \frac{2cd}{c^2-v^2}$$

$$= \frac{2cd}{c^2(1-v^2/c^2)} = \frac{2d}{c(1-v^2/c^2)}$$

The beam of light reflected from A must leave the half silvered plate G along the path AD, so that the resultant of velocity of light and the velocity of ether drift may act in the direction AC as shown in fig (2)

Hence the resultant velocity of light along

$$AC = \sqrt{c^2 - v^2}$$

After reflection at C from the mirror M_1, the light is reflected along CE, so that the resultant of velocity of light and the velocity of ether drift again acts in the direction CA as shown in fig (3).

So the resultant velocity of light along

$$CA = \sqrt{c^2 - v^2}$$

Hence the time taken by the light to go from A to C and back from C to A

$$T_y = \frac{d}{\sqrt{c^2-v^2}} + \frac{d}{\sqrt{c^2-v^2}}$$

$$= \frac{2d}{\sqrt{c^2-v^2}}$$

$$= \frac{2d}{c\sqrt{1-\frac{v^2}{c^2}}}$$

Thus due to ether drift, time taken by light to go from A to B and back again to A will be greater than the time taken by light to go from A to C and back again to A by an amount

$$\Delta T = T_x - T_y = \frac{2d}{c\left(1-\frac{v^2}{c^2}\right)} - \frac{2d}{c\sqrt{1-\frac{v^2}{c^2}}}$$

$$= \frac{2d}{c}\left\{1 + \frac{v^2}{c^2} - 1 - \frac{1}{2}\frac{v^2}{c^2}\right\}$$

$$= \frac{2d}{c}\left\{\frac{v^2}{2c^2}\right\} = \frac{d}{c} \cdot \frac{v^2}{c^2}$$

During this time light travels through a distance

$$C \cdot \Delta T = C\frac{d}{c} \cdot \frac{v^2}{c^2} = \frac{dv^2}{c^2}$$

This is the path difference introduced between two parts of the incident beam. If the whole apparatus is turned through 90^0, the arm AC will become optically longer than the arm AB by an amount $\frac{dv^2}{c^2}$, Hence the effect of rotation of the apparatus is to produce a path difference $\frac{2dv^2}{c^2}$.

The fringe pattern observed in the telescope will, therefore, shift through a number of fringes n given by

$$\frac{2dv^2}{c^2} = n\lambda$$

$$\text{Or} \quad n = \frac{2dv^2}{c^2} \cdot \frac{1}{\lambda}$$

Where λ is the wave length of light used.

To have an observable shift, Michelson & Morley increased the effective value of l up to 11 meters by reflecting the light back and forth several times. Then using values.

l = 11 meters, $v = 3 \, x \, 10^4 m/s$

$C = 3 \, x \, 10^8 \, {}^{m}/_{s}$ & $\lambda = 5.5 \, x \, 10^{-7} meter$

The expected shift is

$$\Delta N = \frac{2lv^2}{c^2\lambda} = \frac{2 \, x \, 11 \, x \, (3 \, x \, 10^4)^2}{(3 \, x \, 10^8)^2 \, x \, (5.5 \, x \, 10^{-7})} = 0.4$$

Or a shift of four tenths a fringe.

Michelson & Morley was extremely surprised to see that there was no shift in the fringe when the interferometer was rotated through 90^0. Thus, the motion of the earth through ether could not be experimentally detected.

Explanation of the Negative results :-

1. **Ether- drag Hypothesis :-** The moving earth completely drags the ether with it so that there is no relative motion between the two and hence the question of shift does not arise. But this explanation was not accepted for two reasons –

(i) It goes against the observed aberration of light from stars.

(ii) Fizeau had shown that a moving body could drag the light waves only partially.

(2) Fitzerald. Lorentz contraction Hypothesis –

Fitzerald & Lorentz independently put an adhoc hypothesis that all material bodies moving through the ether are contracted in the direction of motion by a factor $\sqrt{1-\frac{v^2}{c^2}}$, It is easily seen that such a contraction in the interferometer arm would equalize the times T_x & T_y and no fringe shift would be expected. This explanation also could not be accepted.

Einstein Revolutionary Idea

Einstein in 1905 proposed a new revolutionary idea that the motion through ether is a meaningless concept, only motion relative to a frame of reference has physical significance.

The postulates of special Relativity Theory :-

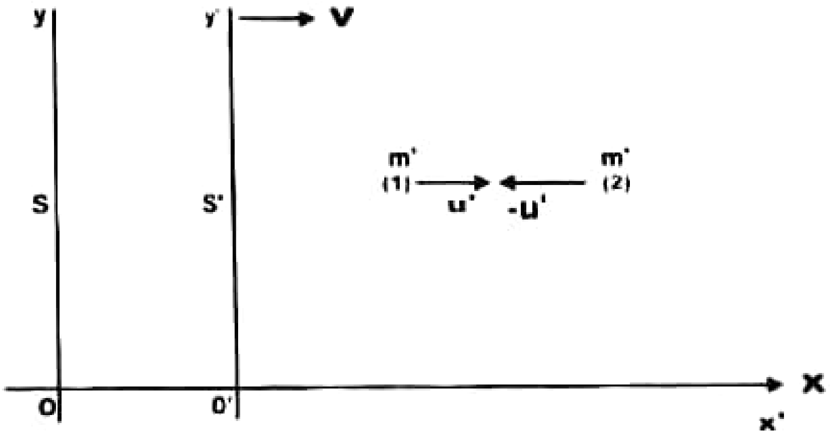
(i) The laws of Physics are the same in all intertial systems. No preffered inertial system exists.

(ii) The speed of light in free space has the same value C in all inertial systems.

Variation of Mass of a Particle with its velocity

According to the classical Newtonian dynamics, the mass of a moving body is constant independent of velocity. But the theory of relativity leads us to a very different conclusion, viz. the variation of mass with velocity for which an expression can be derived as follows.

Let us consider two bodies of mass m' moving in opposite direction along x-axis with velocities u & -u' as observed from frame of reference s'. Let these bodies collide and coalesce in to one body. The body thus formed



will be at rest according to the law of conservation of momentum with respect to S' . If the collisions of the two bodies is observed from frame of reference S, the velocities of the two bodies as observed from S will be given by

$$u_1 = \frac{u'+v}{1+\frac{u'v}{c^2}} \quad \text{And} \quad u_2 = \frac{-u'+v}{1-\frac{u'v}{c^2}} \qquad \text{—(1)}$$

Let m_1 and m_2 be the masses of the two bodies with respect to frame S. Then the body formed when the two bodies coalesce in to each other has a mass (m_1+m_2) by the law of conservation of mass and it moves with the velocity v along the x-axis with respect to S. It is to be noted that this body is at rest with respect to S'. Then by the law of conservation of momentum we have

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v \qquad \text{—(2)}$$

Or $$m_1\left(\frac{u'+v}{1+\frac{u'v}{c^2}}\right) + m_2\left(\frac{-u+v}{1-\frac{u'v}{c^2}}\right) = (m_1 + m_2)v \qquad \text{—(3)}$$

Dividing eq^n (3) throughout by m_2 and simplifying we get

$$\frac{m_1}{m_2} = {1+\frac{u'v}{c^2}}\Big/{1-\frac{u'v}{c^2}} \qquad \text{—(4)}$$

Now let us consider the factor $\left(1-\frac{u_1^2}{c^2}\right)$

Substituting the value of u_1 we have

$$1-\frac{u_1^2}{c^2} = 1-\frac{1}{c^2}\left(\frac{u'+v}{1+\frac{u'v}{c^2}}\right)^2$$

$$= \frac{c^2\left(1+\frac{u'v}{c^2}\right)^2 - (u'+v)}{c^2\left(1+\frac{u'v^2}{c^2}\right)}$$

$$= \frac{\left(1-\frac{v^2}{c^2}\right)(c^2-u'^2)}{c^2\left(1+\frac{u'v}{c^2}\right)^2}$$

$$= \frac{\left(1-\frac{v^2}{c^2}\right)c^2\left(1-\frac{u'^2}{c^2}\right)}{c^2\left(1+\frac{u'v}{c^2}\right)^2}$$

$$\therefore 1 - \frac{u_1^2}{c^2} = \frac{\left(1-\frac{v^2}{c^2}\right)\left(1-\frac{u'^2}{c^2}\right)}{\left(1+\frac{u'v}{c^2}\right)^2} \quad \text{—— (5)}$$

$$\text{Or } \sqrt{1 - \frac{u_1^2}{c^2}} = \frac{\sqrt{\left(1-\frac{v^2}{c^2}\right)\left(1-\frac{u'^2}{c^2}\right)}}{\left(1+\frac{u'v}{c^2}\right)}$$

$$\text{Or } \left(1 + \frac{u'v}{c^2}\right) = \frac{\sqrt{\left(1-\frac{v^2}{c^2}\right)\left(1-\frac{u'^2}{c^2}\right)}}{\sqrt{1-\frac{u_1^2}{c^2}}} \quad \text{—— (6)}$$

$$\text{Similarly } \left(1 - \frac{u'v}{c^2}\right) = \frac{\sqrt{\left(1-\frac{v^2}{c^2}\right)\left(1-\frac{u'^2}{c^2}\right)}}{\sqrt{1-\frac{u_2^2}{c^2}}} \quad \text{—— (7)}$$

Substituting these values in equ^n (4) we have

$$\frac{m_1}{m_2} = \sqrt{1-\frac{u_2^2}{c^2}}\Bigg/\sqrt{1-\frac{u_1^2}{c^2}} \qquad \text{—(8)}$$

If the velocity of the second body as observed with respect to S is zero, i.e $u_2 = 0$ then its mass m_2 can be denoted by m_0. The symbol m_0 gives the mass of a body when it is at rest with respect to the frame of reference being used. Let $u_1 = v$ i.e velocity of the first body with respect to s is v. We can write $m_1 = m$ then equ^n (8) becomes

$$\frac{m}{m_0} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

Or $$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \qquad \text{—(9)}$$

This is the relativistic formula for the variation of mass with velocity, m_0 is called the rest mass that is when the body whose mass is measured is at rest relative to the observer, m is the effective mass that is the mass of the body when it is moving with the velocity v with respect to the observer. Hence as far as the observer is concerned masses of any moving system appear to increase with velocity, becoming infinite when v attains the velocity of light c. This indicates that c is a limiting velocity unattainable by moving material body. When v is small compared with c, $\frac{v^2}{c^2}$ becomes negligible and the mass of the body remains sensibly constant. This law has been directly verified by the experiments of Kaufmann, Bucherer and Guye & Lavanchy on high speed electrons. Many other observations also have confirmed its validity.

Einstein's Mass Energy Retation

As we know, the kinetic energy of a body is measured by the amount of workdone to accelerate the body from its state of rest to the required state of motion at velocity v. If a force F acting on a body displaces it through a distance dx along its direction of application, the workdone by the force = **f.dx**, which is stored by the body as its kinetic energy dE_k. Thus,

$$dE_k = F \,.\, dx \qquad \text{—(1)}$$

From Newton second law of motion the force equals to the rate of change of momentum, i.e

$$F = \frac{d}{dt}(mv)$$

Since both m & v are variable

$$F = m\frac{dv}{dt} + v\frac{dm}{dt} \qquad \text{—(2)}$$

$$\therefore F.\,dx = \left(m.\frac{dv}{dt} + v.\frac{dm}{dt}\right).\,dx$$

$$\therefore Ed_k = \left(m.\frac{dv}{dt} + v.\frac{dm}{dt}\right).\,dx$$

$$= m.\frac{dv}{dt}.\,dx + v.\frac{dm}{dt}.\,dx$$

$$= m.\frac{dx}{dt}.\,dv + v.\frac{dx}{dt}.\,dm$$

$$= m.v\,dv + v^2.\,dm \qquad \text{—(3)}$$

Now according to mass transformation equ^n.

$$m = {m_0}\Big/\sqrt{1-\frac{v^2}{c^2}}$$

Or $m^2 = \frac{{m_0}^2}{1-\frac{v^2}{c^2}}$

Or $m^2 - \frac{m^2v^2}{c^2} = {m_0}^2$

Or $m^2c^2 - m^2v^2 = {m_0}^2c^2$

Or $m^2c^2 = m^2v^2 + {m_0}^2c^2$

Differentiating we get

$$2m\,dm\,c^2 = 2m\,dm\,v^2 + 2v.dv.m^2$$

$$c^2dm = v^2dm + mv\,dv \quad \text{—(4)}$$

From equ^n. (3) and (4) we have

$$d\,E_k = c^2\,dm$$

If the mass of the body changes from m_0 to m and the corresponding change in kinetic energy E_k, we have on integration

$$\int d\,E_k = \int_{m_0}^{m} c^2 dm$$

Or $E_k = c^2\,(m - m_0)$ —(5)

The equ^n (5) shows that the kinetic energy of a moving body is c^2 times the gain in mass. It thus indicates that the increase in mass is an indication and measure of the gain in kinetic energy.

The mass m_0 is often termed as the rest mass of the body and the term m_0c^2 as the rest mass energy of the body. This rest mass energy is regarded as a form of internal energy inherent in the nature of particles constituting the matter. Rewriting equ^n (5) we have

$$mc^2 = m_0c^2 + E_k$$

Or Total energy = Rest mass energy + kinetic energy

Denoting the total energy as E, we have

$$E = mc^2 \qquad \text{—(6)}$$

It can be easily shown with the help of equ^n (5) that if $v \ll c$, the expression for E_k reduces to its classical form

$$E_k = c^2(m - m_0)$$

$$= c^2m - c^2m_0$$

$$= c^2\frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} - c^2m_0$$

$$= c^2m_0\left\{\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right\}$$

$$= c^2 m_0 \left\{ \left(1 - \frac{v^2}{c^2}\right)^{\frac{-1}{2}} - 1 \right\}$$

$$= c^2 m_0 \left\{ 1 + \frac{1}{2}\frac{v^2}{c^2} - 1 \right\}$$

$$= m_0 c^2 + \frac{1}{2}\frac{v^2}{c^2}$$

$$= \frac{1}{2} m_0 v^2$$

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