Dr. Supriya Kumari
Deptt. of Physics,
J. N College, Madhubani

Electromagnetic Theory

Maxwell proposed that if there is an alternating electric field and an alternating magnetic field in mutually perpendicular directions, then the result is flow of energy called the electromagnetic energy in a direction perpendicular to electric and magnetic field. All the vectors form a right handed system. The energy flows in the form of wave called electromagnetic wave.

Maxwell was actually attracted by the fact that an electric current can produce a magnetic field whereas a changing magnetic field can produce electric current. Hence, he argued that electric and magnetic phenomena must be closely related. He studied the behaviour of electric and magnetic field vectors, the electric field intensity vector $\vec{E}$, the magnetic field intensity vector $\vec{H}$, the electric displacement vector $\vec{D}$ and magnetic displacement vector $\vec{B}$. As a consequence of these studies he arrived at the conclusion of propagation of electromagnetic energy.

The field vectors $\vec{E}$, $\vec{H}$, $\vec{D}$ and $\vec{B}$ are related to each other through the following four equations :

$$\vec{\nabla}\cdot\vec{D} = \rho \quad -(1)$$
$$\vec{\nabla}\cdot\vec{B} = 0 \quad -(2.)$$
$$\vec{\nabla}\times\vec{E} = \dot{\vec{B}} \quad -(3.)$$
$$\vec{\nabla}\times\vec{H} = \dot{\vec{D}} + \vec{J} \quad -(4.)$$

These equations are called Maxwell's field equations. In the above equation ρ is volume density of charge and $\vec{j}$ is the electric current density vector. The quantities $\vec{E}$ and $\vec{H}$ are related to $\vec{D}$ and $\vec{B}$ through the relation $\vec{D} = \varepsilon\vec{E}$ and $\vec{B} = \mu\vec{H}$ where ε and μ are the dilectric constant and permeability of the medium in which electric and magnetic fields are produced. Thus, $\vec{j} = \sigma\vec{E}$, $\vec{D} = \varepsilon\vec{E}$ and $\vec{B} = \mu\vec{H}$ are called the material equations or constitutive equations. Naturally the propagation of electromagnetic wave affected by the properties of the medium.

Maxwell's Field equations:

(1.) The first equation is the vector form of Gauss's theorem in electrostatics. It may be stated that the total flux through a surface enclosing by unit volume is equal to the charge ~~density~~ enclosing by it, i.e the volume density of charge, which is $\text{Div}\cdot\vec{D} = \rho$ or $\nabla\cdot\vec{D} = \rho$.

(2.) The second equation is the mathematical representation of the fact that there exist no free magnetic pole. North and South magnetic poles always occur together. Hence, if a closed surface encloses n free north poles then there are n south poles also producing equal and opposite flux through the surface so that the resulting flux is zero. Hence

$$\text{Div } \vec{B} = 0 \text{ or } \nabla \cdot \vec{B} = 0.$$

(3.) The third equation is the vector form of Faraday's law of electromagnetic induction. The quantity $\nabla \times \vec{E}$ gives the workdone in carrying unit +(ve) charge around a circuit which is called the electromotive force. This is given by the rate of change of magnetic flux linked with the area having the circuit as the boundary, i.e

$$\nabla \times \vec{E} = -\vec{B}$$

(4.) The fourth equation is a modified version of Ampere's theorem. $\nabla \times \vec{H}$ when converted to line integral by Stoke's theorem $\nabla \times \vec{H}$ ~~wh~~ gives the workdone in carrying unit north pole around an electric current. This quantity gives the ~~value~~ of electric current in Amperes. The term $\vec{D}$, which gives the rate of change of $\vec{D}$ with time was added to Ampere's theorem by Maxwell. It is called displacement current density and arises due to change in electric displacement in the medium with time.

i.e $\nabla \times \vec{H} = \vec{J} + \dot{\vec{D}}$.