

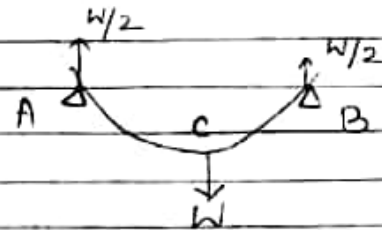
Bending of beam & Cantilever. (For $\sigma = I(H)$ & $\sigma = I(S)$)

Q:- Derive an expression for the bending of a beam supported at the two ends and loaded in the middle. Describe an experiment to determine γ by bending.

When the bar beam is supported at the ends and loaded in the middle.

Let a beam be supported on two knife edges at the two ends A & B and let it be loaded in the middle at C with a load W.

The reaction at each knife edge will clearly be in the upward direction.



Since the middle part of the beam is horizontal, the beam may be considered as equivalent

to two inverted cantilevers, fixed at C, the bending being produced by the load $W/2$ acting upwards at A and B. If, therefore, l be the length of the beam AB, the length of the cantilever is $l/2$ and the elevation of A or B above C or what is the same thing, the depression of C below A & B is given by

$$y = \frac{W/2 (l/2)^3}{3\gamma I} = \frac{Wl^3}{48\gamma I}$$

If the beam be of rectangular cross-section,

$$I = \frac{bd^3}{12} \text{ and therefore, for such a beam}$$

$$y = \frac{Wl^3}{48\gamma} \cdot \frac{12}{bd^3} = \frac{Wl^3}{4\gamma bd^3} \quad \text{--- (1)}$$

If the beam be of circular cross-section; we have

$$I = \frac{\pi r^4}{4}, \text{ so that for such a beam}$$

$$y = \frac{Wl^3}{48\gamma} \cdot \frac{4}{\pi r^4} = \frac{Wl^3}{12\gamma \pi r^4} \quad \text{--- (2)}$$

Determination of γ by bending of a beam

The beam is supported horizontally and symmetrically on two parallel knife edges, a known distance l apart. A hanger with a hook is arranged in the middle of the beam. By loading the rod at the centre with a load increasing in equal steps and then decreasing the load in the same equal steps, the mean depression y for a certain load can be found out. The depression y at the centre of the load can be found out. The depression y at the centre of the beam accurately with the help of a spherometer or travelling microscope. The breadth b and thickness d are also measured. Knowing all the quantities we can calculate γ by using the relation

$$\gamma = \frac{Wl^3}{4Ybd^3}$$