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Q:- Derive the Fresnel's formula for the reflected and refracted amplitudes.

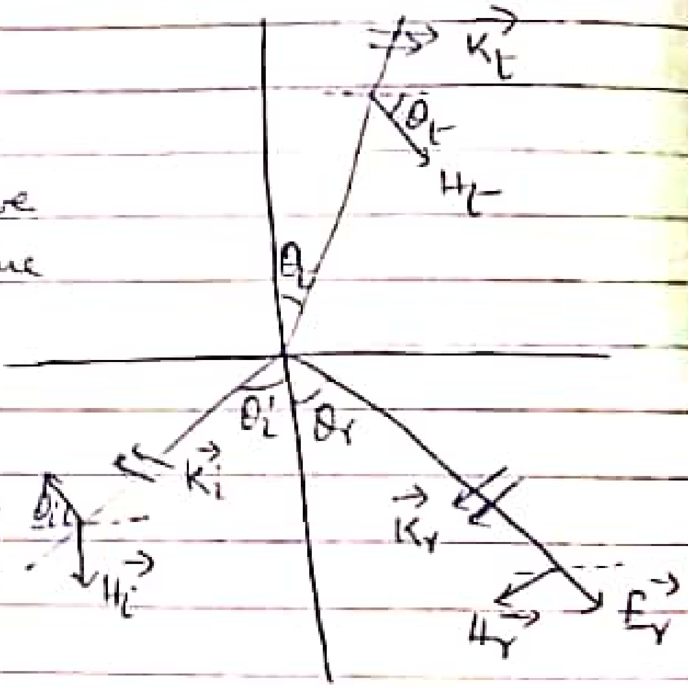
Ans:- Fresnel's equations: The equations relating the amplitudes of the reflected and transmitted waves with that of incident wave are known as Fresnel's eqns or formula.

To derive the coefficient of the reflection and transmission when a plane polarised wave is incident on an interface of two dielectric, we consider the case when

(i) electric vector E ⊥ to the plane of incidence
(ii) electric vector E ∥ to the plane of incidence.

Case I

(1) Let us now consider the reflection and refraction of a linearly polarised plane wave with its el. vector ⊥ to the plane of incidence, the reflection is assumed to occur at the interface of two dielectrics. The mag. field vectors and the propagation vectors are incident as in the fig.



The ele. vector will be along y-axis and all directed to the plane of figure and are given by

$$\vec{E_i} = \hat{y} E_{i_0} e^{i[\vec{k_i}\cdot\vec{r} - \omega t]}$$

$$\vec{E_r} = \hat{y} E_{r_0} e^{i[\vec{k_r}\cdot\vec{r} - \omega t]}$$

and $\vec{E_t} = \hat{y} E_{t_0} e^{i[\vec{K_t}.\vec{r} - \omega t]}$

where $\vec{E_i}, \vec{E_r}, \vec{E_t}$ denote the el. fields associated with incident, reflected and refracted or transmitted waves respectively. Since y-axis is tangential to the surface (inter-face), the components of $\vec{E}$ must be continuous across the interface. Consequently,

$$\vec{E_{i_0}} + \vec{E_{r_0}} = \vec{E_{t_0}} \quad \text{--- (1)}$$

Since the displacement vector $\vec{D}$ has no component normal to the incidence, the continuity of normal component of $\vec{D}$ will not give any eqn. The direction of the mag. fields are also shown in fig. 2 They lie in the plane of incidence and are given by

$$\vec{H_i} = H_{i_0} e^{i[\vec{K_i}.\vec{r} - \omega t]}$$
$$= \left(\frac{\vec{K_i} \times \vec{E_{r_0}}}{\omega \mu_1}\right) \cdot e^{[i(\vec{K_i}\vec{r} - \omega t)]}$$

$$\vec{H_r} = H_{r_0} e^{i[\vec{K_r}.\vec{r} - \omega t]}$$
$$= \left(\frac{\vec{K_i} \times \vec{E_{r_0}}}{\omega \mu_1}\right) e^{i[\vec{K_r}.\vec{r} - \omega t]}$$

$$\vec{H_t} = \vec{H_{t_0}} e^{i[\vec{K_t}.\vec{r} - \omega t]}$$
$$= \left(\frac{\vec{K_t} \times \vec{E_{t_0}}}{\omega \mu_2}\right) e^{i[\vec{K_t}.\vec{r} - \omega t]} \quad \text{--- (2)}$$

Since H lies in the plane of incidence. Since $\vec{k_i} \perp$ to $\vec{E_{i_0}}$, the magnitude H_{i_0} is simply $\left(\frac{k_i E_{i_0}}{\omega \mu_1}\right)$. Similarly for the $\vec{H_{i_0}}$ & $\vec{H_{r_0}}$, it is clear from fig that z-components of the mag. field to to be continuous, we must have

$$H_{i_0} \cos\theta_i - H_{r_0} \cos\theta_i = H_{t_0} \cos\theta_t \quad \text{--- (3)}$$

or $$\frac{k_i}{\omega \mu_1}\left(E_{i_0} - E_{r_0}\right)\cos\theta_i = \frac{k_t E_{t_0} \cos\theta_t}{\omega \mu_2} \quad \text{--- (4)}$$

Putting the expression for E_{t_0} from (1) we get

$$\frac{k_i}{\omega \mu_1}\left[E_{t_0} - E_{r_0}\right]\cos\theta_i = \frac{k_t}{\omega \mu_2}\left[E_{t_0} + E_{r_0}\right]\cos\theta_t$$

Rearranging we get the amplitude reflection coeff. for normal ~~pola~~ polarisation.

$$\left(\vec{r_n}\right) = \frac{E_{r_0}}{E_{i_0}} = \frac{\frac{k_i}{\omega\mu_1}\cos\theta_i - \frac{k_t}{\omega\mu_2}\cos\theta_t}{\frac{k_i}{\omega\mu_1}\cos\theta_i + \frac{k_t}{\omega\mu_2}\cos\theta_t}$$

$$= \frac{\sqrt{\epsilon_1/\mu_1}\cos\theta_i - \sqrt{\epsilon_2/\mu_2}\cos\theta_t}{\sqrt{\frac{\epsilon_2}{\mu_1}}\cos\theta_i + \sqrt{\frac{\epsilon_2}{\mu_2}}\cos\theta_t}$$

$$= \frac{\sin\theta_t \cos\theta_i - \sin\theta_i \cos\theta_t}{\sin\theta_t \cos\theta_i + \sin\theta_i \cos\theta_t}$$

or $$r_n = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)} \quad \text{--- (5)}$$

This is the required expⁿ for amplitude reflectivity for normal polarisation.

Further the amplitude transmission coeff. (transmissivity) for normal polarisation is given by

$$t_N = \frac{E_{to}}{E_{io}} = 1 + \frac{E_{ro}}{E_{io}} = 1 - \frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)}$$

$$= \frac{\sin(\theta_i + \theta_t) - \sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)}$$

$$= \frac{2 \sin\theta_t + \cos\theta_i}{\sin(\theta_i + \theta_t)} \quad \text{——} \quad (6)$$