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Fresnel formula
(continued...)

(2). E vector ∥ to the plane of incidence:

Now we consider the el. vector ($\vec{E}$) lie in the plane of incidence. The mag. vector ($\vec{H}$) are along y-axis. Z-components of the el. vector represents a tangential component which should be continious across the surface. Thus

$$E_{iz} + E_{rz} = E_{tz} \quad — (7)$$

or

$$-E_{i_0} \cos\theta_i + E_{r_0} \cos\theta_i = E_{t_0} \cos\theta_t$$

For the continuity conditions to be satisfied, this can be also be written in the following form

$$[E_{i_0} - E_{r_0}] \cos\theta_i = E_{t_0} \cos\theta_t \quad — (8).$$

Further for normal ~~incidence~~ component of D must be continious ($D = \epsilon E$).

$$\epsilon_1 E_{ix} + \epsilon_2 E_{tx}$$

or $\epsilon_1 [E_{i_0} + E_{r_0}] \sin\theta_i = \epsilon_2 E_{t_0} \sin\theta_t$ —— (9)

$$\epsilon_1 [E_{i_0} + E_{r_0}] \sin\theta_i = \epsilon_2 \sin\theta_t \frac{[E_{i_0} - E_{r_0}] \cos\theta_i}{\cos\theta_t}$$

Rearranging we get the amplitude refl. coeff. for parallel component polarisation.

$$r_p = \frac{E_{r_0}}{E_{i_0}} = \frac{\epsilon_2 \sin\theta_t \cos\theta_i - \epsilon_1 \sin\theta_i \cos\theta_t}{\epsilon_2 \sin\theta_t \cos\theta_i + \epsilon_1 \sin\theta_i \cos\theta_t} \quad \text{—— (13)}$$

amplitude transmission coeff. for $\parallel$ polarisation

$$t_p = \frac{E_{t_0}}{E_{i_0}} = \frac{2\epsilon_1 \sin\theta_t \cos\theta_i}{\epsilon_2 \sin\theta_t \cos\theta_i + \epsilon_1 \cos\theta_t \sin\theta_i} \quad \text{—— (14)}$$

But we know that for optical frequency

$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{n_2}{n_1}$$

~~But we know~~ considering this fact and solving we get

$$r_p = \frac{\sin\theta_i \cos\theta_i - \sin\theta_t \cos\theta_t}{\sin\theta_i \cos\theta_i + \sin\theta_t \cos\theta_t}$$

$$= \frac{\sin 2\theta_t - \cos 2\theta_t}{\sin 2\theta_i + \cos 2\theta_t}$$

$$= \tan(\theta_i - \theta_t) / \tan(\theta_i + \theta_t) \quad \text{—— (15)}$$

and

$$t_p = \frac{2\cos\theta_i \sin\theta_t}{\sin(\theta_i + \theta_t)} \quad \text{—— (16)}$$

Equ^n (15) & (16) are also called Fresnel's formula or equ^n for the parallel polarisation (ie electric vector containing the component ∥ to the plane of incidence).

These two kinds of di-vector are therefore, independent of one another. Equ^n (15) & (16) lead to the following conditions :-

I :- For normal incidence $\theta_i = 0$, $\theta_t = 0$ | $n = \frac{n_2}{n_1}$

$$\left.\begin{aligned} r_n &= -\frac{(n-1)}{n+1} \\ t_n &= \frac{2}{n+1} \\ r_p &= \frac{n-1}{n+1} \\ t_p &= \frac{2}{n+1} \end{aligned}\right\} \quad (17)$$

This means the distinction between ∥ to ⊥ components disappears now.

II :- when $n_1 = n_2$, no reflection of wave occurs

when $n_2 = n_1$, $\theta_t = \theta_i$

we get $r_p = 0$, $t_p = 1$

Thus there is no reflection of the wave, when the second medium has the same refractive indices as the first medium has.