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D-1 (Hons & Sub). Paper-II

Law of Equipartition of Energy.

The Maxwell's law in terms of velocity components is

$$dN = N\left(\frac{m}{2\pi KT}\right)^{3/2} e^{-m(u^2+v^2+w^2)/2KT} \, du\, dv\, dw$$

Since each of the three components must have the same velocity distribution, the fraction of molecules with velocities between u and $u+du$, irrespective of other components, is given by

$$\frac{dN(u)}{N} = \left(\frac{m}{2\pi KT}\right)^{1/2} e^{-mu^2/2KT} \, du.$$

The average kinetic energy of a molecule moving in the x-direction is given by

$$\bar{E}_x = \frac{1}{N}\int_{-\infty}^{+\infty} E_x \, dN(u)$$

$$= \frac{1}{2} m \left(\frac{m}{2\pi KT}\right)^{1/2} \int_{-\infty}^{+\infty} e^{-mu^2/2KT} u^2 du.$$

Putting $\frac{m}{2KT} = b$, we have

$$\bar{E}_x = \frac{1}{2} m \left(\frac{b}{\pi}\right)^{1/2} \int_{-\infty}^{+\infty} e^{-bu^2} u^2 du.$$

The value of the integral is $\frac{1}{2}\sqrt{\frac{\pi}{b^3}}$. Therefore,

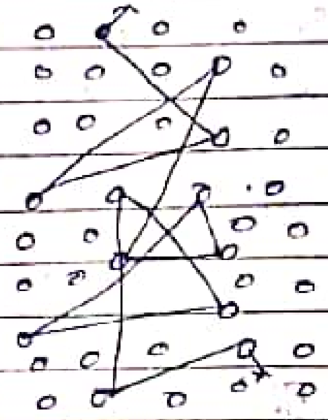
$$\bar{E}_x = \frac{1}{4} mb = \frac{1}{2} KT.$$

In the same way, $\bar{E}_y = \bar{E}_z = \frac{1}{2} KT$.

Mean Free Path :

According to the kinetic theory, the molecules of a gas are constantly moving in all directions and with various speeds.

They frequently collide with one another when their speeds and directions change. They frequently collide with one another when their speeds and directions change. They do not exert any force upon one another except at collision. Therefore, they move in straight lines with constant speeds between two successive collisions. Hence, if we watch a particular molecule, it will be found to have short zig-zag paths of different lengths. These are called the "free paths" of the molecule and their mean is called the "mean free path".



(Fig 1)

The mean free path is the average distance travelled by a molecule between two successive collisions with other molecules.

Expressions for Mean Free path :-

Let us assume that all the molecules, except one, are at rest. Let d be the diameter of a molecule. Clearly a collision will take place when the moving molecule approaches a molecule such that the centres of the two molecules are within a distance d of one another. Alternatively, we may describe the collision by regarding the moving molecule as having a diameter 2d and all the other molecules as

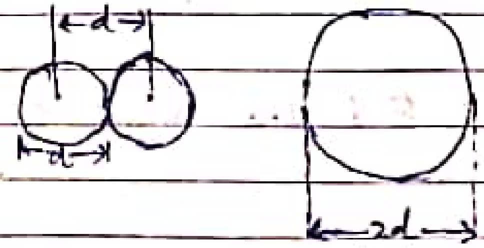


Fig (2)

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point - particles in Fig (3)

Let us now follow a single molecule of equivalent diameter $2d$ as it moves through a gas of point-particles.

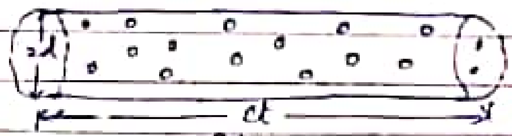

Fig (3)

If $\bar{c}$ be the average speed of this molecule, then in time t it will sweep out a cylinder of cross-sectional area πd^2 and of length $\bar{c}t$. In time t this molecule will collide with every other molecule whose centre lies in this cylinder. Thus if there are n molecules per unit volume, the number of collisions made in time t by the equivalent molecule

$$= n \times \text{volume of the cylinder} = n \times \pi d^2 \bar{c} t$$

The mean free path λ is simply the average distance between two successive collisions. Hence

$$\lambda = \frac{\text{total distance covered in time } t}{\text{number of collisions in time } t}$$

$$= \frac{\bar{c}t}{n \times \pi d^2 \bar{c} t} = \frac{1}{n \pi d^2}$$

If we give up the assumption that all the other molecules are at rest, the expression will be modified as

$$\lambda = \frac{1}{\sqrt{(2)}\, \pi n d^2}$$

Thus the mean free path is inversely proportional to the number of molecules per unit volume (n) which is proportional to the density of the gas. Thus the mean free path varies inversely as the density and, therefore, inversely as the pressure of the gas.