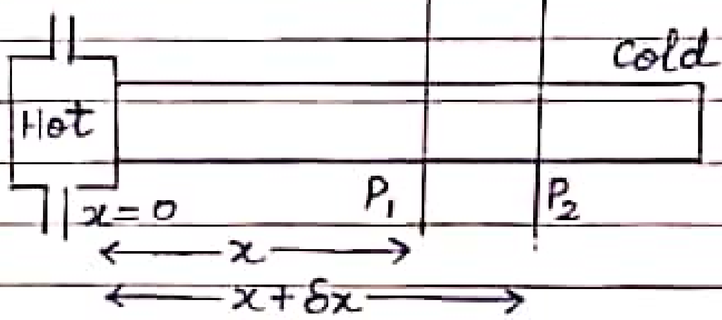
Dr. Supriya kumari
Dept. of Physics.

Conduction of Heat
(Continued ...)

Fundamental Equation of Conduction : Fourier's equation for heat conduction :

Let us consider a long metal bar of uniform area of cross-section A heated at one end. Heat flows along the length of the bar. Let us suppose that the bar lies along the x-axis of a set of coordinates, whose origin lies at the hot end. In the beginning the temperature at various points along the bar gradually rises but finally a steady state is reached at which the temp. at any point of the bar becomes steady. If the bar is exposed to the surroundings, a portion of the heat passing across any cross section of the bar escapes out from the sides and hence a smaller & smaller amount of heat passes across successive section of the bar.



Let us consider two parallel planes P_1 and P_2 perpendicular to the length of the bar at a distance x and $x+\delta x$ from the hot end. Let θ be the excess of temperature above the surroundings at the plane P_1 and $\frac{d\theta}{dx}$ the temperature gradient, then

excess of temperature above the surroundings at

$P_2 = \left(\theta + \frac{d\theta}{dx}\delta x\right)$.

$\therefore$ Temp. gradient at $P_2 = \frac{d}{dx}\left(\theta + \frac{d\theta}{dx}\delta x\right)$

$$= \frac{d\theta}{dx} + \frac{d^2\theta}{dx^2}\delta x.$$

If Q_1 and Q_2 be the amount of heat that enter the plane P_1 and leave the plane P_2 per second respectively, then

$$Q_1 = -KA\frac{d\theta}{dx}$$

and, $Q_2 = -KA\left(\frac{d\theta}{dx} + \frac{d^2\theta}{dx^2}.\delta x\right)$

Hence, amount of heat gained per second by the element of thickness δx is given by

$$Q = Q_1 - Q_2 = -KA\frac{d\theta}{dx} - \left\{-KA\left(\frac{d\theta}{dx} + \frac{d^2\theta}{dx^2}.\delta x\right)\right\}$$

$$= KA\cdot\frac{d^2\theta}{dx^2}\cdot\delta x \quad \text{—} \quad (3)$$

Before the steady state is reached this heat is partly used in raising its temperature. If $\frac{d\theta}{dt}$ is the rate of increase in temp. of the element, ρ the density and S the specific heat of ~~the~~ the material, then heat required to raise the temperature of the element at the above stated rate

$$= A\cdot\delta x\cdot\rho S\frac{d\theta}{dt} \quad \text{—} \quad (4)$$

If P is the perimeter of the element, then surface area of the element $= P \cdot \delta x$.

If E be the radiating power of the surface and θ be the average excess of temp. of the element, the heat radiated per second by the element

$$= E \cdot P \cdot \delta x \cdot \theta.$$

Hence,

$$KA \cdot \frac{d^2\theta}{dx^2} \cdot \delta x = A \cdot \delta x \cdot \rho s \frac{d\theta}{dt} + E \cdot P \delta x \theta$$

$$\text{or, } \frac{d^2\theta}{dx^2} = \frac{\rho s}{K} \cdot \frac{d\theta}{dt} + \frac{EP}{KA} \cdot \theta \quad \text{——— (5)}$$

This is the general equation representing the unidirectional rectilinear flow of heat and is called Fourier's differential equation for one dimensional flow of heat.