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Stefan-Boltzmann law

Based on experimental results of Tyndall and of Dulong and Petet, Stefan emperically deduced that total radiation from any heated body is proportional to the fourth power of its absolute temperature. Boltzmann gave a theoretical proof of the law based on thermodynamical consideration. He showed that the law applies strictly to emission from a black body. The law is therefore known as Stefan-Boltzmann law and may be formally enunciated as follows: If a black body at absolute temperature T be surrounded by another black body at absolute temp. T_0, the amount of energy E lost per second per unit area of the former is

$$E = \sigma (T^4 - T_0^4) \quad \text{—— (1)}$$

For proving this law, let us consider radiation in a black body chamber and try to apply thermodynamical laws to the radiation. Let u denote the energy density of radiation inside the enclosure, V its total volume and P the pressure. Then both u and P are simply functions of the absolute temperature. The total energy of radiation is

$$U = uV \quad \text{—— (2)}$$

From Maxwell electromagnetic theory of light,

$$P = \frac{u}{3} \quad \text{—— (3.)}$$

Let us suppose that a small amount of radiation dQ flows into the enclosure from outside due to which energy changes by dU and volume by dV. Then from thermodynamics we have

$$dQ = dU + PdV$$

$$= d(uV) + \frac{1}{3} u\, dV$$

$$= u\, dV + V du + \frac{1}{3} u dV.$$

$$= \frac{4}{3} u dV + V du \quad \text{—— (4.)}$$

If dS be the change in entropy of radiation, then $dQ = T.dS$

$$\therefore \; TdS = \frac{4}{3} u dV + V du.$$

or $$dS = \frac{4}{3} \cdot \frac{u}{T} dV + \frac{V}{T} du \quad \text{—— (5.)}$$

Now dS is a perfect differential,

$$\therefore \; dS = \left(\frac{\partial S}{\partial u}\right)_V du + \left(\frac{\partial S}{\partial V}\right)_V dV \quad \text{—— (6.)}$$

Comparing eq^n (5) & (6.), we have

$$\left(\frac{\partial S}{\partial u}\right)_V = \frac{V}{T} \text{ and } \left(\frac{\partial S}{\partial V}\right)_u = \frac{4}{3}\frac{u}{T}.$$

$$\therefore \frac{\partial}{\partial V}\left(\frac{\partial S}{\partial u}\right)_V = \frac{\partial}{\partial u}\left(\frac{\partial S}{\partial V}\right)_u$$

or $$\frac{\partial}{\partial V}\left(\frac{V}{T}\right) = \frac{\partial}{\partial u}\left(\frac{4}{3}\frac{u}{T}\right)$$

or. $$\frac{1}{T} = \frac{4}{3}\left(\frac{1}{T} - \frac{1}{T^2}u\cdot\frac{\partial T}{\partial u}\right).$$

or, $$\frac{4}{3}\frac{u}{T^2}\frac{\partial T}{\partial u} = \frac{1}{3T}$$

or $$\frac{4}{T}\cdot u\cdot\frac{\partial T}{\partial u} = 1$$

or $$\frac{\partial u}{u} = 4\cdot\frac{\partial T}{T}$$

Integrating we have :

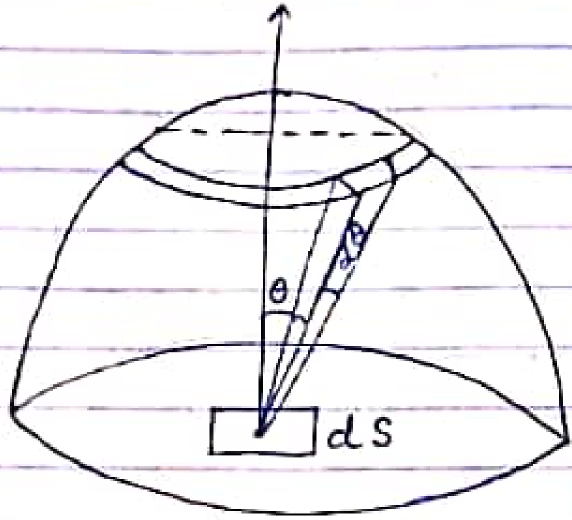
$\log u = 4\log T + \log A$, where A is constant

or, $u = AT^4$ —— (7)

This gives energy density of radiation in the enclosure. This may be expressed in terms of emissive power. Let us consider a small element dS with a sphere of unit ~~no~~ radius surrounding it. The energy travels in all

directions with velocity of light c.
The energy density of any solid angle $d\omega$ is

$$= \frac{u c \, d\omega}{4\pi}$$



The amount of radiation crossing dS in a direction making an angle θ with the normal to dS is

$$= \frac{u c \, d\omega \cos\theta}{4\pi}$$

The solid angle between θ and dθ is

$$d\omega = 2\pi \sin\theta \, d\theta.$$

So, the radiation crossing dS is

$$= \frac{u c \, dS \sin\theta \cdot \cos\theta \, d\theta}{2}.$$

And the radiation crossing ~~dS~~ s per unit area is

$$= \frac{uc}{2} \sin\theta \cdot \cos\theta \, d\theta.$$

Integrating, the rate of emission per unit area is

$$E = \frac{1}{2} uc \int_0^{\pi/2} \sin\theta \cos\theta \, d\theta = \frac{uc}{4} \quad \text{—(8)}$$

Hence, from (5.) and (7) we have

$$E = \frac{1}{4} AC\, T^4$$

$$= \sigma T^4 \qquad \text{—— (9.)}$$

where $\sigma = \frac{AC}{4}$ is called Stefan's constant.

Now, when this body is surrounded by another body at temp T_0, it will also emit radiation equal to σT_0^4 which falls on the body at temp T, so the net loss due to radiation per unit area of the body at temp T, per unit time is

$$E = \sigma (T^4 - T_0^4) \qquad \text{—— (10.)}$$