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Thermodynamic (or Absolute or Kelvin's work) Scale of temperature

According to the Carnot's theorem, the efficiency of a reversible engine is independent of the working substance and depends only on the two temperatures between which it is working. Thus there is a property which absolutely depends on temperature and nothing else. Taking this hint, Lord Kelvin defined a temperature scale which does not depend upon the properties of any particular substance. This scale is called absolute scale of temperature. Since this is based on thermodynamics hence it is also known as thermodynamic scale of temperature or sometimes as Kelvin's scale work.

Let a reversible engine take in Q_1 heat from a source at temperature θ_1 and give out Q_2 heat to sink at temp θ_2, where θ_1 and θ_2 have ~~ma~~ been measured on any scale. The efficiency of this engine is $\eta = 1 - \frac{Q_2}{Q_1}$, which depends on θ_1 and θ_2 only we may write,

$$\eta = 1 - \frac{Q_2}{Q_1} = f(\theta_1, \theta_2), \quad \text{—— (1)}$$

where f is an unknown function. From this one may also say that $\frac{Q_1}{Q_2}$ must be a function of θ_1 and θ_2 only. From eqn (1) we have

$$\frac{Q_1}{Q_2} = \frac{1}{1 - f(\theta_1, \theta_2)} = F(\theta_1, \theta_2) \quad \text{—— (2)}$$

where F is some other unknown fn.

Similarly, for a reversible engine taking in Q_2 heat at temp θ_2 and giving out heat Q_3 at temp θ_3, we have

$$\frac{Q_2}{Q_3} = F(\theta_2, \theta_3) \quad \text{———— (3)}$$

The heat Q_2 given out by the first engine is taken in by the second. Thus, both engines working together, form a third engine which takes in Q_1 heat at temp θ_1 and gives out heat Q_3 at temp θ_3, where

$$\frac{Q_1}{Q_3} = F(\theta_1, \theta_3) \quad \text{———— (4)}$$

We have from the above,

$$\frac{Q_1}{Q_3} = \frac{Q_1}{Q_2} \times \frac{Q_2}{Q_3} = F(\theta_1, \theta_2) \times F(\theta_2, \theta_3)$$

or $F(\theta_1,\theta_3) = F(\theta_1,\theta_2) \times F(\theta_2,\theta_3)$ ——— (5)

from the equation (5) it is evident that it does not contain θ_2 on the left side, clearly, function F should be so chosen that θ_2 disappears from RHS also. This is possible if

$$F(\theta_1,\theta_2) = \frac{\varphi(\theta_1)}{\varphi(\theta_2)} \text{ and } F(\theta_2,\theta_3) = \frac{\varphi(\theta_2)}{\varphi(\theta_3)} \quad \text{——— (6)}$$

Here φ is another unknown fn of temp. Hence, we have from (5) and (6),

$$F(\theta_1,\theta_3) = \frac{\varphi(\theta_1)}{\varphi(\theta_2)} \times \frac{\varphi(\theta_2)}{\varphi(\theta_3)} = \frac{\varphi(\theta_1)}{\varphi(\theta_3)} \quad \text{——— (7)}$$

Therefore, for any Reversible engine,

$$\frac{Q_1}{Q_2} = \frac{\varphi(\theta_1)}{\varphi(\theta_2)} \quad \text{——— (8)}$$

We know $Q_1 > Q_2$. Hence the function $\varphi(\theta_1) > \varphi(\theta_2)$, when $\theta_1 > \theta_2$. It means that fun $\varphi(\theta)$ increases as ~~θ increases~~ temperature increases. Hence it can be used to measure temperatures. Let us denote the value $\varphi(\theta)$ by T on the new scale. The relation (8) then becomes

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2} \quad \text{——— (9)}$$

The eqn (9) is used to define a new scale of temp T, which is called the thermodynamic or Kelvin Scale. This scale does not depend upon the properties of any particular substance for eqn (9) is universally true. The ratio of any two temperatures on this scale is equal to the ratio of the heats taken in an rejection by an engine working reversibly between the two temperatures.

The zero of this scale ($T=0$) is that temp at which $Q_2 = 0$ and hence $W = Q_1$. Thus all the heat taken by the engine has been converted into work and the efficiency of the engine is unity. T can not be less than this, ie negative, for if it were so, Q_2 would be negative which implies that the engine would be drawing heat ~~from~~ both from the source and the sink. This is impossible from the second law and hence $\tau = 0$, is the lowest temp conceivable. This is the thermodynamic definition of the absolute zero of temp. The zero of the scale having been determined the size of degrees is now to be fixed. In conformity with the common practice, the interval between the freezing point and boiling point of water at normal pressure is divided into 100 equal parts on this scale. That is,

$$\frac{Q_{steam}}{Q_{ice}} = \frac{\tau_{ice} + 100}{\tau_{ice}} \quad \text{——— (10)}$$

The thermodynamic scale is thus completely defined & fixed. Now, let us see how this scale can be realised in practice as the Carnot reversible engine does not exist. For a Carnot engine using a perfect gas as the working substance,

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2} \quad \text{——— (11)}$$

, where T_1 and T_2 are the temps measured on perfect gas scale. Hence, from eqn. (9)

$$\frac{\tau_1}{\tau_2} = \frac{T_1}{T_2} \quad \text{——— (12)}$$

obviously, when $T_1 = 0$, $\tau_1 = 0$ and

$$\frac{\tau_{ice} + 100}{\tau_{ice}} = \frac{T_{ice} + 100}{T_{ice}}$$

or $\tau_{ice} = T_{ice}$

Thus the thermodynamic scale and perfect gas scale are identical. Hence, thermodynamic scale in practice is realised by perfect gas scale

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