

Electric field & Potential

Electric field: Electric field at any point may be defined as the force experienced by a unit positive charge placed at that ~~time~~ point.

$$\vec{E} = \frac{\Delta \vec{F}}{\Delta q} \text{ Newton per coulomb}$$

Potential: The electric potential at any point in an electric field is defined as the work done in taking a unit positive charge from infinitely to that point against the electrical forces.

$$E = -\frac{dV}{dx}$$

Electric dipole: An electric dipole is a system of two point charges of equal magnitude but opposite nature separated by infinitesimally small distance.

Dipole moment: An electric dipole moment (P) is defined as a vector whose magnitude is $2ql$ and which is directed from negative charge to the positive charge, i.e. $P = 2ql \hat{p}$ where \hat{p} is the unit vector along the axis of the dipole.

Potential at a point due to dipole :

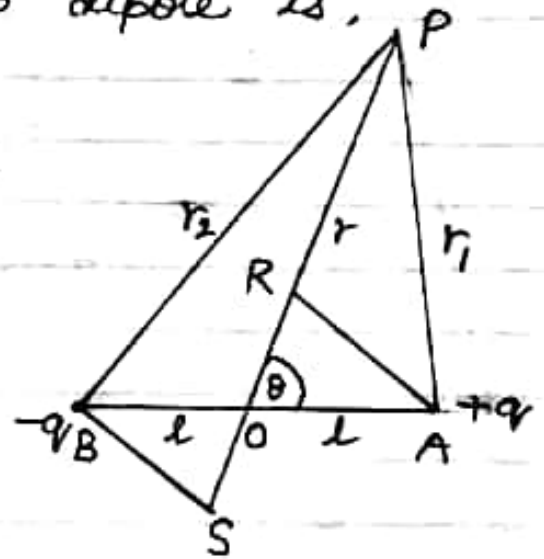
Let us consider an electric dipole consisting of $-q$ and $+q$ separated by a distance $2l$. Then the potential at $P(r, \theta)$ due to dipole is,

$$V = \frac{1}{4\pi\epsilon_0} \left(\frac{q}{r_1} - \frac{q}{r_2} \right) = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

$$= \frac{q}{4\pi\epsilon_0} \left(\frac{1}{r - l\cos\theta} - \frac{1}{r + l\cos\theta} \right)$$

$$= \frac{q}{4\pi\epsilon_0} \frac{2l\cos\theta}{r^2 - l^2\cos^2\theta}$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{p\cos\theta}{r^2} \quad (\text{neglecting } l^2\cos^2\theta \text{ which is small}).$$

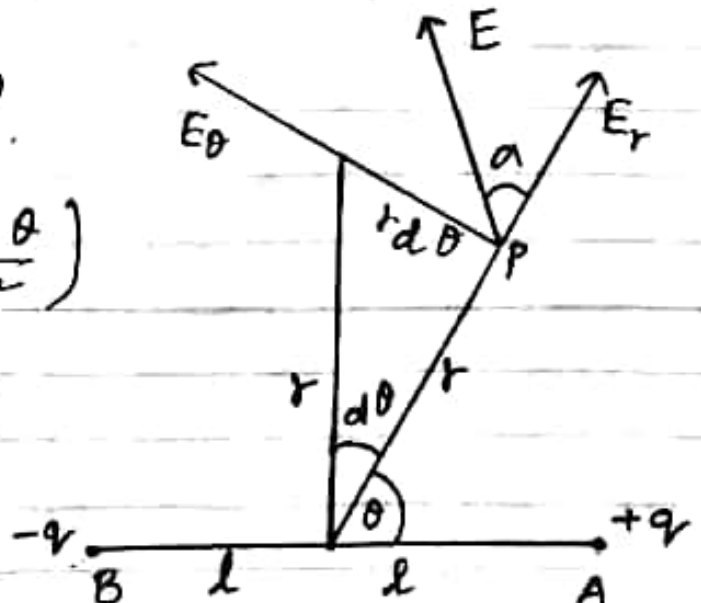


Intensity at any point due to a dipole :

$$\text{Potential at } P = \frac{1}{4\pi\epsilon_0} \cdot \frac{p\cos\theta}{r^2}$$

$$E_r = -\frac{dV}{dr} = -\frac{d}{dr} \cdot \frac{1}{4\pi\epsilon_0} \left(\frac{p\cos\theta}{r^2} \right)$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{2p\cos\theta}{r^3}$$



3.

$$\text{and } E_{\theta} = -\frac{dv}{rd\theta} = -\frac{1}{r} \frac{d}{d\theta} \cdot \frac{1}{4\pi\epsilon_0} \left(\frac{P \cos \theta}{r^2} \right)$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{P \sin \theta}{r^3}$$

$$\therefore E = \sqrt{E_r^2 + E_{\theta}^2} = \frac{1}{4\pi\epsilon_0} \sqrt{\frac{4P^2 \cos^2 \theta + P^2 \sin^2 \theta}{r^6}}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{P}{r^3} \cdot \sqrt{1 + 3 \cos^2 \theta}$$

$$\tan \alpha = \frac{E_{\theta}}{E_r} = \frac{4\pi\epsilon_0}{4\pi\epsilon_0} \cdot \frac{P \sin \theta / r^3}{2P \cos \theta / r^3}$$

$$= \frac{1}{2} \tan \theta$$

$$\therefore \alpha = \tan^{-1} \left(\frac{1}{2} \tan \theta \right)$$
