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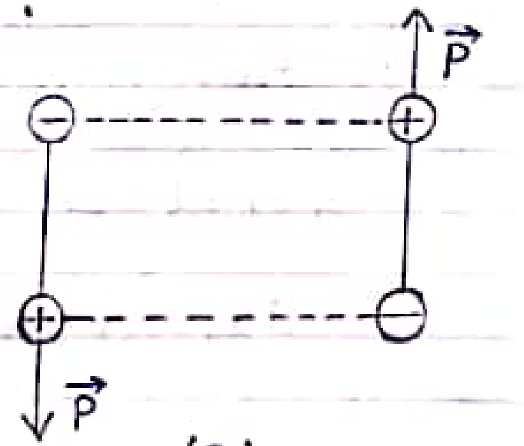
Electric field & Potential (Continued)

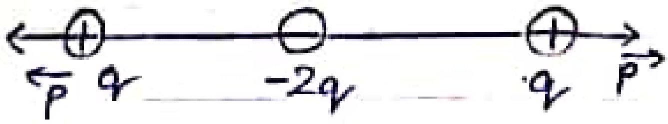
Electric quadripole : A quadripole is a pair of two identical dipoles (and hence a system of four point charges) having their dipoles in opposite directions and seperated by an infinitesimally small distance.

Fig. (a) shows a quadripole in general and fig. (b) shows a linear quadripole.



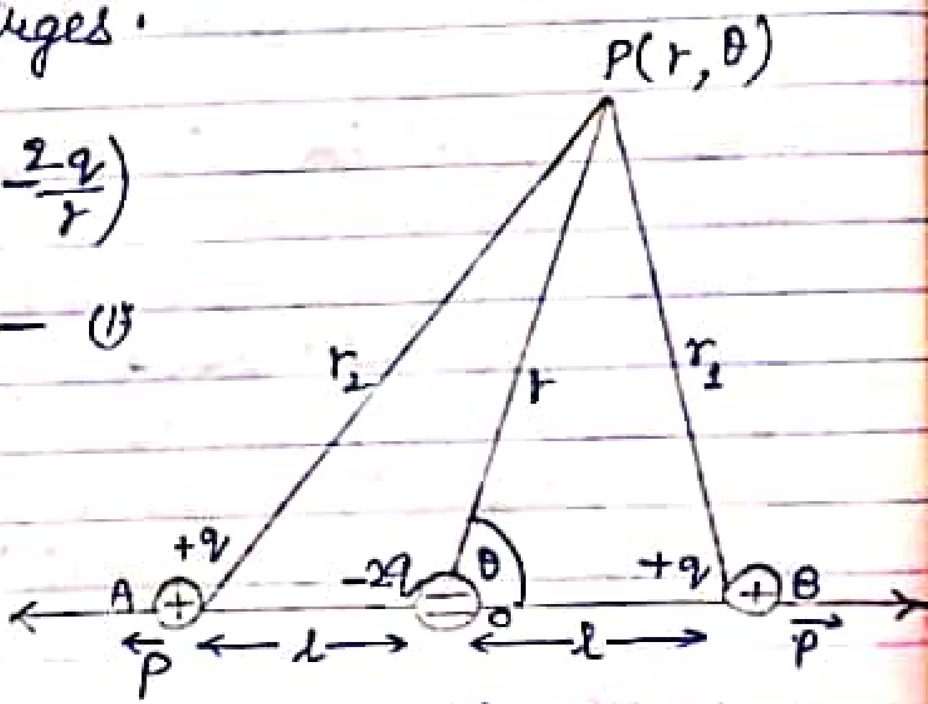
(a.)

Potential & field due to a linear quadripole :-



(b.)

Let us consider a linear quadripole consisting of two dipoles OA and OB, having their negative charges coincident at O and positive charges seperated from O by l. Let us suppose P be a point at a distance r in a direction θ with the axis of the quadripole ($r \gg l$). The potential at P is the sum of the potentials due to the charges.



$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_1} + \frac{q}{r_2} - \frac{2q}{r}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_1} + \frac{1}{r_2} - \frac{2}{r}\right) \quad \text{--- (1)}$$

From fig.,

$$r_1^2 = r^2 + l^2 - 2rl\cos\theta$$

$$\therefore r_1 = (r^2 + l^2 - 2rl\cos\theta)^{1/2} = r\left(1 + \frac{l^2}{r^2} - 2\frac{l}{r}\cos\theta\right)^{1/2}$$

$$\therefore \frac{1}{r_1} = \frac{1}{r}\left(1 - \frac{2l}{r}\cos\theta + \frac{l^2}{r^2}\right)^{-1/2} = \frac{1}{r}\left\{1 - \left(2\frac{l\cos\theta}{r} - \frac{l^2}{r^2}\right)\right\}^{-1/2}$$

$$= \frac{1}{r}\left\{1 + \frac{1}{2}\left(\frac{2l\cos\theta}{r} - \frac{l^2}{r^2}\right) + \frac{3}{8}\left(\frac{2l\cos\theta}{r} - \frac{l^2}{r^2}\right)^2 + \frac{5}{16}\left(\frac{2l}{r}\cos\theta - \frac{l^2}{r^2}\right)^3 + \dots\right\}.$$

Now, collecting terms upto third power of $\left(\frac{l}{r}\right)$ and neglect the rest as $r >> l$.

$$\therefore \frac{1}{r_1} = \frac{1}{r}\left\{1 + \frac{1}{2}\left(\frac{2l\cos\theta}{r} - \frac{l^2}{r^2}\right) + \frac{3}{8}\left(\frac{4l^2\cos^2\theta}{r^2} - \frac{4l^3\cos\theta}{r^3}\right) + \frac{5}{16}\left(\frac{8l^3}{r^3}\cos^3\theta\right).\right.$$

$$= \frac{1}{r}\left\{1 + \frac{l\cos\theta}{r} - \frac{l^2}{2r^2}(1 - 3\cos^2\theta) + \frac{l^3}{2r^3}(5\cos^3\theta - 3\cos\theta)\right\}.$$

We note that r_2 is given by the same formula, simplify the place of θ is taken by $(\pi - \theta)$. Since $\cos(\pi-\theta) = -\cos\theta$.

$$\therefore \frac{1}{r_2} = \frac{1}{r}\left\{1 - \frac{l}{r}\cos\theta - \frac{l^2}{2r^2}(1-3\cos^2\theta) - \frac{l^3}{2r^3}(5\cos^3\theta - 3\cos\theta)\right\}.$$

$$\therefore \frac{1}{r_1} + \frac{1}{r_2} = \frac{1}{r}\left\{2 - \frac{l^2}{r^2}(1-3\cos^2\theta)\right\} = \frac{2}{r} - \frac{l^2}{r^3}(1-3\cos^2\theta).$$

$$\therefore V = \frac{q}{4\pi\varepsilon_0}\left\{\frac{2}{r} - \frac{l^2}{r^3}(1-3\cos^2\theta) - \frac{2}{r}\right\}.$$

$$= \frac{q}{4\pi\varepsilon_0}\frac{l^2}{r^3}(3\cos^2\theta - 1).$$

(i) When the point is on the axial line then $\theta = 0$, $\cos\theta = 1$

$$\therefore V = \frac{1}{4\pi\varepsilon_0}\frac{2ql^2}{r^3}$$

(ii) When the point is on equatorial line, $\theta = 90^\circ$, $\cos\theta = 0$. $V = \dfrac{-ql^2}{4\pi\varepsilon_0 r^3}$.

Electric intensity

$$E_r = -\frac{dv}{dr} = \frac{1}{4\pi\epsilon_0}\frac{-d}{dr}\left\{\frac{ql^2}{r^3}(3\cos^2\theta - 1)\right\}.$$

$$= \frac{1}{4\pi\epsilon_0}\frac{3qL^2}{r^4}(3\cos^2\theta - 1).$$

and $E_\theta = -\frac{dv}{r d\theta} = -\frac{1}{r}\cdot\frac{dv}{d\theta}$.

$$= -\frac{1}{r}\cdot\frac{d}{d\theta}\left\{\frac{1}{4\pi\epsilon_0}\cdot\frac{ql^2}{r^3}(3\cos^2\theta - 1)\right\}.$$

$$= \frac{1}{4\pi\epsilon_0}\cdot\frac{6ql^2}{r^4}\sin\theta\cos\theta.$$

$$\therefore E = \sqrt{E_r^2 + E_\theta^2}..$$

$$= \frac{1}{4\pi\epsilon_0}\cdot\frac{3ql^2}{r^4}\sqrt{1 - 2\cos^2\theta + 5\cos^4\theta}.$$

Scalar Potential : The scalar function V whose 'del operation' gives the electric field is called the scalar potential or electrostatic potential. The negative sign in front of 'del' is necessary in order that the electric field intensity shall point towards a decrease in potential.

$$E = \cancel{\vec{\nabla}} . \; - \nabla \vec{V}$$