

Boolean Algebra

The circuits in digital computers follow the logic of mind. Hence symbolic logic, invented by Boole for solving logical problems, can be applied in the analysis and design of digital circuits. This logic is a binary or two valued logic, and resembles ordinary algebra in many respects. Hence this logic is also called Boolean algebra.

Boolean algebra permits only two values or states for a variable. In logic, these two states represent 'true' and 'false' and in circuits, they represent 'on' and 'off' or the 'cut off' & 'saturation' states of an electric device. The two permitted states of Boolean algebra are usually represented by 0 & 1.

Only three operations are employed on the variables in Boolean algebra. These operations are

- (1) the OR addition represented by (+) sign.
- (2) the AND multiplication represented by cross (X), or dot (.) sign.

(3.) the NOT operation represented by a bar over a variable.

(1) OR Addition :- If A & B are two variables then $Y = A + B$ are written in Boolean algebra:

(i) When $A = 0$, $B = 0$, $Y = 0$.

$$\text{Hence } Y = A + B = 0 + 0 = 0$$

(ii) When $A = 0$, $B = 1$, $Y = 1$.

$$\text{Hence, } Y = A + B = 0 + 1 = 1.$$

(iii) When $A = 1$, $B = 0$, $Y = 1$.

$$\text{Hence, } Y = A + B = 1 + 0 = 1.$$

(iv) When $A = 1$, $B = 1$, $Y = 1$,

$$\text{Hence, } Y = A + B = 1 + 1 = 1.$$

(2) AND Multiplication :-

(i) When $A = 0$, $B = 0$, $Y = 0$

$$\text{Hence, } Y = A \cdot B = 0 \cdot 0 = 0$$

(ii) When $A = 0$, $B = 1$, $Y = 0$

$$\text{Hence, } Y = A \cdot B = 0 \cdot 1 = 0.$$

(iii) When $A = 1$, $B = 0$, $Y = 0$

$$\text{Hence, } Y = A \cdot B = 1 \cdot 0 = 0.$$

(iv) When $A = 1$, $B = 1$, $Y = 1$

$$\text{Hence, } Y = A \cdot B = 1 \cdot 1 = 1.$$

(3) The NOT operation :-

If $A = 0$ then $\bar{A} = 1$ & if $A = 1$,
 $\bar{A} = 0$ thus $Y = \bar{A}$.

Theorems of Boolean Algebra :-

As in ordinary algebra, the commutative, associative & distributive laws are valid for Boolean algebra.

Commutative laws : $A + B = B + A$

$$A \cdot B = B \cdot A$$

Associative laws : $A + (B + C) = (A + B) + C$

$$A(B \cdot C) = (A \cdot B) \cdot C$$

Distributive laws : $A(B + C) = AB + AC$