Dr. Supriya Kumari
Deptt. of Physics
R.N. College, Madhubani.

D-2 (H) Paper-IV

State & prove De Morgan's theorem :-

Theorem 1 :- The complement of the sum of two or more variables is equal to the product of the complements of the variables.

Theorem 2 :- The complement of the product of two or more variables is equal to the sum of the complements of the variables.

For two variables A & B, these theorems are written, in Boolean notations as follows:

$$\overline{A+B} = \bar{A}\cdot\bar{B}$$

and

$$\overline{A\cdot B} = \bar{A}+\bar{B}.$$

To prove $\overline{A+B} = \bar{A}\cdot\bar{B}$;

Since each variable can have a value either 0 or 1, the following four cases arise :

(i) When $A=0$, $B=0$, $\overline{A+B} = \overline{0+0} = \bar{0} = 1$,
and. $\bar{A}\cdot\bar{B} = \bar{0}\cdot\bar{0} = 1\cdot 1 = 1$.
Hence $\overline{A+B} = \bar{A}\cdot\bar{B}$.

(ii) When $A=0$, $B=1$, $\overline{A+B} = \overline{0+1} = \bar{1} = 0$,
and $\bar{A}\cdot\bar{B} = \bar{0}\cdot\bar{1} = 1\cdot 0 = 0$.
Hence, $\overline{A+B} = \bar{A}\cdot\bar{B}$.

(iii) When $A=1$, $B=0$, $\overline{A+B} = \overline{1+0} = \overline{1} = 0$,
and $\bar{A}\cdot\bar{B} = \bar{1}\cdot\bar{0} = 0\cdot 1 = 0$
Hence, $\overline{A+B} = \bar{A}\cdot\bar{B}$.

(iv) When $A=1$, $B=1$, $\overline{A+B} = \overline{1+1} = \overline{1} = 0$,
and $\bar{A}\cdot\bar{B} = \bar{1}\cdot\bar{1} = 0\cdot 0 = 0$
Hence, $\overline{A+B} = \bar{A}\cdot\bar{B}$.

<u>To prove $\overline{A\cdot B} = \bar{A} + \bar{B}$</u> :-

(i) When $A=0$, $B=0$, $\overline{A\cdot B} = \overline{0\cdot 0} = \bar{0} = 1$
and $\bar{A}+\bar{B} = \bar{0}+\bar{0} = 1+1 = 1$.
Hence, $\overline{A\cdot B} = \bar{A}+\bar{B}$.

(ii) When $A=0$, $B=1$, $\overline{A\cdot B} = \overline{0\cdot 1} = \bar{0} = 1$,
and $\bar{A}+\bar{B} = \bar{0}+\bar{1} = 1+0 = 1$
Hence. $\cancel{A+B}$ $\overline{A\cdot B} = \bar{A}+\bar{B}$.

(iii) When $A=1$, $B=0$, $\overline{A\cdot B} = \overline{1\cdot 0} = \bar{0} = 1$
and $\bar{A}+\bar{B} = \bar{1}+\bar{0} = 0+1 = 1$.
Hence $\overline{A\cdot B} = \bar{A}+\bar{B}$.

(iv) When $A=1$ and $B=1$, $\overline{A\cdot B} = \overline{1\cdot 1} = \bar{1} = 0$,
and $\bar{A}+\bar{B} = \bar{1}+\bar{1} = 0+0 = 0$.
Hence, $\overline{A\cdot B} = \bar{A}+\bar{B}$.