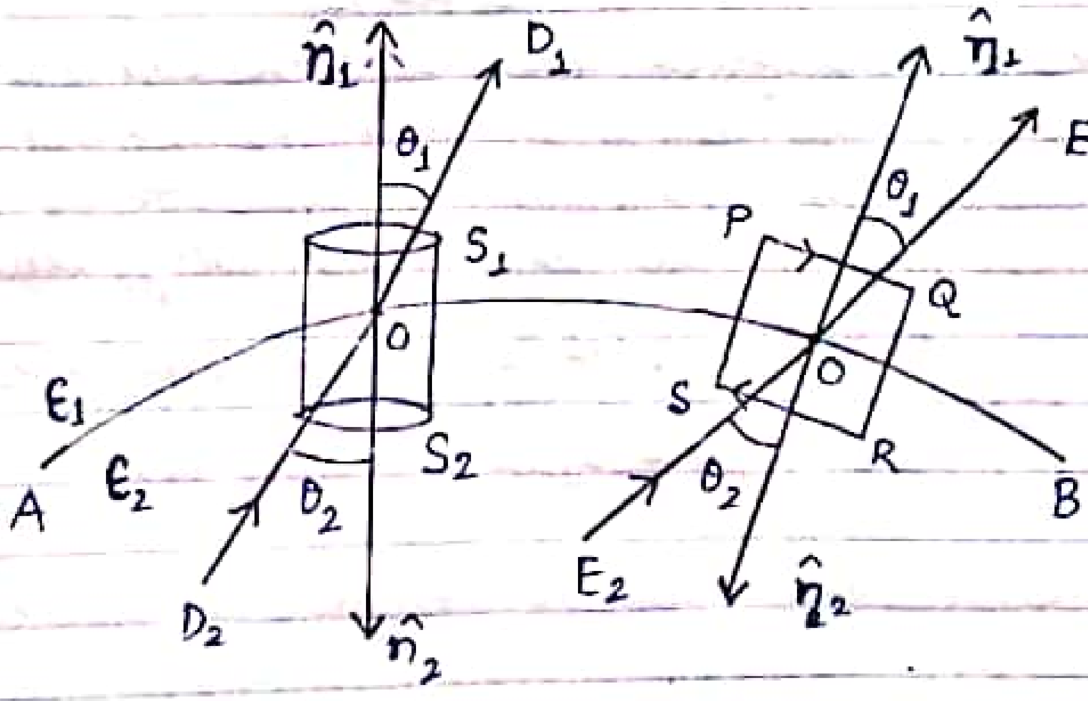
Dr. Supriya Kumari
Deptt. of Physics
J.N. College, Madhubani

D-1(H) Paper - II
D-2(S)

Boundary Conditions at the interface of two dielectrics

When there is a change of media the field vectors $\vec{D}$ & $\vec{E}$ pass from one dielectric medium into another satisfying certain conditions. These conditions are known as boundary conditions.

Let AB represent a small portion of the boundary between two media of permitivities ε_1 and ε_2. The media are assumed to be homogeneous and isotropic. Let us consider an area dS on the boundary, so small that its curvature may be neglected. Let D_1 & D_2 be the electric induction vectors in the media on either side of dS, making angles θ_1 and θ_2 with the normal to dS.



To find the boundary condition for D, let us apply Gauss theorem to a small pill box which intersects the area ds on the boundary and of height very small compared to the diameter of the base. Assuming the volume of the box very small, so that the free charge enclosed by the pill box can be taken as free charge on area ds of the boundary. Thus we have

$$\int D \cdot ds = \int_{c} D \cdot ds + \int_{S_1} D_1 . ds + \int D_2 \, ds = q$$

$$0 + (D_1 . n_1 + D_2 n_2) ds = \sigma ds$$

$$\text{or, } D_1 \cos\theta_1 - D_2 \cos\theta_2 = \sigma .$$

Here the flux through the curved surface is neglected as this area is vanishingly small. In majority of cases there is no free charge on the boundary, hence

$$D_1 \cos\theta_1 - D_2 \cos\theta_2 = 0 \text{ or, } D_1 \cos\theta_1 = D_2 \cos\theta_2$$

$$\text{or, } D_1 n = D_2 n \quad \text{—— (1.)}$$

Therefore the normal component of the

electric displacement is the same on both sides of the boundary of two media of different dielectrics or the normal component is continuous across the boundary having no free charge.
The boundary condition for E is found by considering the work done in taking unit charge round a small rectangle of length dl and of negligible height, with its longest sides parallel to the surface of separation, one in each medium. The work done in taking a unit charge around the rectangle PQRS must vanish, otherwise an infinite amount of work could be obtained by repeating the process indefinitely, hence

$$\oint E\, dl = \int_P^Q E_1\, dl + \int_Q^R E\, dl + \int_R^S E_2\, dl + \int_S^P E \cdot dl = 0$$

$$\text{or, } E_1 \sin\theta_1\, dl - E_2 \sin\theta_2\, dl = 0$$

$$\text{or, } E_1 \sin\theta_1 = E_2 \sin\theta_2 \quad \text{or } E_{1t} = E_{2t} \quad \text{(2.)}$$

Here contribution of short sides QR and SP have been neglected as it is vanishingly small. Therefore the tangential components of the electric intensities are the same on the two sides of the boundary surface or the tangential component of the electric field is continuous at the boundary between two dielectrics.

From eq^n (1.) & (2.) we have:

$$\frac{E_1 \sin\theta_1}{D_1 \cos\theta_1} = \frac{E_2 \sin\theta_2}{D_2 \cos\theta_2}$$

or
$$\frac{E_1 \sin\theta_1}{\varepsilon_1 E_1 \cos\theta_1} = \frac{E_2 \sin\theta_2}{\varepsilon_2 E_2 \cos\theta_2}.$$

$$\therefore \frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_1}{\varepsilon_2} \quad \text{---} \quad (3.)$$

Thus D & E are bent according to this formula.