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For Deg II Chem Hons, Paper – III &
Deg II sub courses

Entropy change in an ideal gas

Let us consider that one mole of an ideal gas has volume V when its temperature is T and Pressure is P

Suppose that δQ is the amount of heat given to this gas reversibly Then its entropy change will be given by

$$dS = \frac{\delta Q_{rev}}{T} \quad \text{---(1)}$$

We know from 1st Law of Thermodynamics that $\delta Q_{rev} = dE + \delta w_{maximum}$

We know that $\delta w_{maximum} = P.dV$

Thus $\delta Q_{rev} = dE + PdV$

$$\text{or} \quad dS = \frac{dE + PdV}{T} = \frac{dE}{T} + \frac{PdV}{T} \quad \text{---(2)}$$

We know that $C_V = \left(\frac{\partial E}{\partial T}\right)_V \therefore dE = C_V dT$

For 1 mole ideal gas $PV = RT \therefore P = \frac{RT}{V}$

Substituting the value of dE and P in equation (II)

$$dS = \frac{C_V dT}{T} + \frac{RT\, dV}{TV}$$

Integrating the above equation we get

$$\int dS = C_V \int \frac{dT}{T} + R \int \frac{dV}{V}$$

or $S = C_v \log T + R \log V + S_0$ — (3)

where S_0 = Integration Constant

Thus entropy for state 1 (Initial state) can be given as

$S_1 = C_v \log T_1 + R \log V_1 + S_0$ — (4)

Entropy for state 2 (Final state) can be given as

$S_2 = C_v \log T_2 + R \log V_2 + S_0$ — (5)

Hence $\Delta S = C_v \log \frac{T_2}{T_1} + R \log \frac{V_2}{V_1}$ — (6)

Another form :—

We know that

$S = C_v \log T + R \log V + S_0$

We know that $C_p - C_v = R$

$\therefore C_v = C_p - R$

We know that $PV = RT$ (For 1 mole ideal gas)

$\therefore V = \frac{RT}{P}$

Substituting these values in the above equation we get

$S = (C_p - R) \log T + R \log \frac{RT}{P} + S_0$

$= C_p \log T - R \log T + R \log R + R \log T - R \log P + S_0$

or $= C_p \log T - R \log P + (R \log R + S_0)$

$= C_p \log T - R \log P + S_0'$

where S_0' is another constant.

When a process undergoes a change from state 1 to state 2 its entropy will be given by,

$$S_1 = C_p \log T_1 - R \log P_1 + S_0'$$

$$S_2 = C_p \log T_2 - R \log P_2 + S_0'$$

$\therefore$ Entropy change $(\Delta S) = S_2 - S_1$

$$\Delta S = C_p \log \frac{T_2}{T_1} - R \log \frac{P_2}{P_1}$$

Different situations:—

(1) Entropy change for a process at constant pressure will be given by

$$\Delta S = C_p \log \frac{T_2}{T_1} \quad [\text{As } P_1 = P_2]$$

(2) Entropy change for a process at constant volume will be given by

$$\Delta S = C_v \log \frac{T_2}{T_1} \quad [\text{As } V_1 = V_2]$$

(3) Entropy change for a process at constant temperature will be given by

$$\Delta S = -R \log \frac{P_2}{P_1}$$

$$\Delta S = +R \log \frac{V_2}{V_1}$$

Here $T_1 = T_2$