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Difference between two specific heats

Q. Derive an expression for the difference between the two specific heats of a perfect gas.

Ans.- Let us consider a gram molecule of a gas having a volume V at a pressure P and temp. T K contained in a cylinder closed by a piston as shown in fig.

The heat required to raise the temperature by a small amount dT at constant volume is given by

$$dQ_v = C_v \, dT \qquad \text{— (i)}$$

where C_v is the gram molecular specific heat at constant volume (in units of work).

If, instead of the volume, the pressure is kept constant then heat required to raise the temperature by a small amount dT at constant pressure is given by

$$dQ_p = C_p \, dT \qquad \text{— (ii)}$$

where C_p is the gram molecular specific heat at constant pressure (in units of work).

As the pressure remains constant, the volume increases and work is done by the gas. If A is the area of the piston which moves outward due to the expansion of the gas by a distance dx, then

Work done $dW = PA\,dx = P\,dV$.

According to first law of thermodynamics for an isochoric process i.e, when the process takes place at constant volume, we have

$$dQ_v = dU_v + dW_v$$

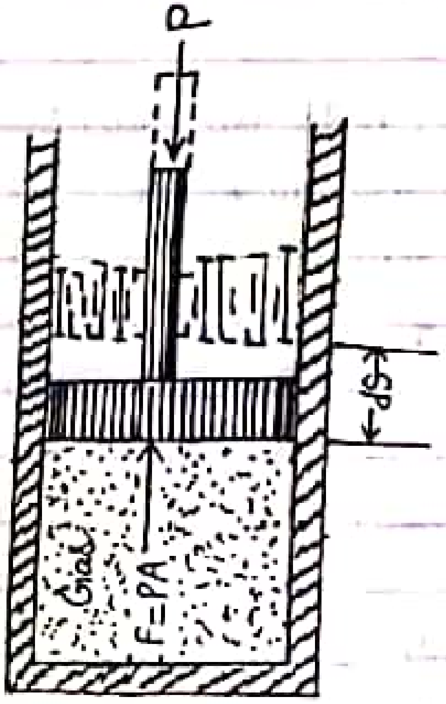

Fig.

As the volume remains constant,

$$dW_v = P\,dV = 0$$

$$\therefore dQ_v = dU_v \quad \text{—(iii)}$$

From (i) and (iii), we have $dU_v = C_v\,dT$ —(iv)

For an isobaric process i.e, when the process takes place at constant pressure

$$dW_p = P\,dV_p$$

and according to first law of thermodynamics

$$dQ_p = dU_p + P\,dV_p \quad \text{— (V)}$$

From (ii) and (v.), we have

$$C_p dT = dU_p + P dV_p$$

As the internal energy of an ideal gas only depends upon temperature and the temperature difference in the two processes, one isochoric (at constant volume) and the other isobaric (at constant pressure) is the same.

$$\therefore dU_p = dU_v$$

Hence from (iv) and (vi), we have

$$C_p dT = C_v dT + P dV_p \quad -(vii)$$

For an ideal gas $PV = RT$. If the pressure is kept constant, then

$$P dV_p = R dT.$$

substituting in (vii), we have

$$(C_p - C_v) dT = R dT \quad -(viii)$$

or $$C_p - C_v = R$$

Equation (viii) holds good for any number of moles of a perfect gas. According to this eq^n it is clear that specific heat at constant pressure is always greater than the specific heat at constant volume.