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Lorentz Transformation eqn (Continued)
D-1 (11 & 5) Paper I

Eqn. (1), (3), (4) & (5) constitute a co-ordinate transformation in space & time which satisfies the first postulate of relativity.

The value of k can be evaluated from second postulate of relativity.

Let us imagine that at the time $t = t' = 0$ when the origin O' coinsides with the origin O a spherical pulse of light leaves the common origin of S and S'. As the velocity of light is invarient each observer sees a spherical wave expanding outwards with the speed c in his own system as measured by his own meter stick & clock. For the observer O the distance travelled by light in a certain time along x axis is given by

$$x = ct \quad \text{—— (6)}$$

and for O' it is given by

$$x' = ct' \quad \text{—— (7)}$$

Substituting the value of x' and t' from eqn (1) & (5) in eqn (7) we have

$$k\,(x - vt) = c\left[kt + \left(\frac{1-k^2}{kv}\right)x\right]$$

or $\quad kx - \left(\frac{1-k^2}{kv}\right)cx = ckt + kvt$

or $\quad x\left\{k - \left(\frac{1-k^2}{kv}\right)c\right\} = ckt + kvt$

or $\quad x = \dfrac{ckt + kvt}{k - \left(\frac{1-k^2}{kv}\right)c}$

$$= \frac{ckt\left(1+\frac{v}{c}\right)}{k\left[1 - \left(\frac{1-k^2}{k^2}\right)\frac{c}{v}\right]}$$

$$= \frac{ct\left(1+\frac{v}{c}\right)}{1-\left(\frac{1}{k^2}-1\right)\frac{c}{v}}$$

$$x = \frac{x\left(1+v/c\right)}{1-\left(\frac{1}{k^2}-1\right)\frac{c}{v}}$$

$$\therefore \quad 1+\frac{v}{c} = 1 - \frac{c}{v}\cdot\frac{1}{k^2} + \frac{c}{v}$$

$$\frac{c}{v}\cdot\frac{1}{k^2} = \frac{c}{v} - \frac{v}{c} = \frac{c^2-v^2}{vc}$$

$$\frac{1}{k^2} = \frac{c^2-v^2}{c^2} = 1-\frac{v^2}{c^2}$$

$$\frac{1}{k} = \sqrt{1-v^2/c^2}$$

or $\quad k = \dfrac{1}{\sqrt{1-v^2/c^2}}$ ——— (8)

Substituting the values of K in eqns (1) & (5) we have

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}$$

$$y' = y$$

$$z' = z$$

and $$t' = \frac{t - vx/c^2}{\sqrt{1 - v^2/c^2}}$$

These are Lorentz transformation eqns. In order to transform measurements from S' to S we have to replace v by −v. Thus the inverse Lorentz transformation eqns are

$$x = \frac{x' + vt'}{\sqrt{1 - v^2/c^2}}$$

$$y = y'$$

$$z = z'$$

$$t = \frac{t' + vx'/c^2}{\sqrt{1 - v^2/c^2}}$$

Two significant aspects of Lorentz transformation eqn are

1) The measurement of time as well as position depen

depends upon the frame of reference of the observer so that two events which occur simultaneously in one frame of reference need not be simultaneous when viewed from another.

(2.) If the relative velocity v of the frame S' relative to S is very small as compared to the velocity of light c, the Lorentz relativistic transformation reduced to the ordinary Galilean eq^n. i.e

$$x' = x - vt$$

$$y' = y$$

$$z' = z$$

and $$t' = t.$$