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Gauss's theorem of divergence (continued....)
Q-1 (ii) Paper I

So that, denoting the x-component of $\vec{F}$ at the centre of the face by $F_x(P_1)$, we have

flux through the left face of the cube $= -F_x(P_1)\,dy\,dz$

Similarly, flux through the right face of the cube
$$= F_x(P_2)\,dy\,dz,$$
the component $F_x(P_2)$ now being positive.

Taking $F_x(P_2) = F_x(P_1) + \frac{\partial F_x}{\partial x}dx$, we have

flux through the right face of the cube
$$= \left(F_x(P_1) + \frac{\partial F_x}{\partial x}dx\right)dy\,dz$$

$\therefore$ Flux through the left and right faces of the cube
$$= \left[F_x(P_1) + \frac{\partial F_x}{\partial x}dx\right]dy\,dz - F_x(P_1)\,dy\,dz$$
$$= \frac{\partial F_x}{\partial x}dx\,dy\,dz$$

Similarly, flux through top and bottom faces of the cube
$$= \frac{\partial F_y}{\partial y}dx\,dy\,dz$$

and flux through the remaining faces of the cube
$$= \frac{\partial F_z}{\partial z}dx\,dy\,dz$$

Therefore, flux through all the faces of the cube is given by the sum of all these. So that,
$$\iint_{\text{Surface of cube}} \vec{F}\,ds = \left(\frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}\right)dx\,dy\,dz.$$
where ds is the area of the surface of the cube.

Now $\frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} = \vec{\nabla}.\vec{F}$ or $\text{div}\,\vec{F}$

and $dx\,dy\,dz = dV$, volume of the cube

We, therefore, have $\iint_{\text{Surface of cube}} \vec{F}\,ds = (\vec{\nabla}.\vec{F})\,dV$

Therefore, summing up the fluxes through all the elementary cubes,
$$\iint_S \vec{E}\,ds = \iiint_V (\vec{\nabla}.\vec{F})\,dV = \iiint_V (\text{div}\,\vec{F})\,dV,$$
which is Gauss Theorem.