

Maxwell's Thermodynamical Relations,  
[Continued....]

(2) Maxwell's first thermodynamical rel<sup>n</sup>.  
Let us take the volume  $v$  and entropy  $S$   
as independent variables, then

$$S = x \text{ and } v = y$$

$$\therefore \frac{\partial S}{\partial x} = 1 \text{ and } \frac{\partial v}{\partial y} = 1 \text{ also}$$

$$\frac{\partial S}{\partial y} = 0 \text{ and } \frac{\partial v}{\partial x} = 0$$

Substituting these values in (vii), we get

$$\left(\frac{\partial T}{\partial x}\right)_y \left(\frac{\partial S}{\partial y}\right)_x = 0 \text{ and } \left(\frac{\partial P}{\partial y}\right)_x \left(\frac{\partial v}{\partial x}\right)_y = 0$$

$$\therefore \left(\frac{\partial T}{\partial y}\right)_x = - \left(\frac{\partial P}{\partial x}\right)_y$$

Replacing  $y$  by  $v$  and  $x$  by  $S$ , we have

$$\left(\frac{\partial T}{\partial v}\right)_S = - \left(\frac{\partial P}{\partial S}\right)_v \quad \text{--- (viii)}$$

This is known as Maxwell's first thermodynamical relation.

(ii) Maxwell's second thermodynamical relation.

Let us take the temperature  $T$  and volume  $v$  as independent variables, then

$$T = x \quad \text{and} \quad v = y$$
$$\left(\frac{\partial T}{\partial y}\right)_x = 0 \quad \text{and} \quad \frac{\partial v}{\partial y} = 1$$

$$\text{also } \frac{\partial T}{\partial x} = 0 \quad \text{and} \quad \frac{\partial v}{\partial x} = 0$$

Substituting these values in (vii), we have

$$\left(\frac{\partial T}{\partial y}\right)_x \left(\frac{\partial s}{\partial x}\right)_y = 0, \quad \left(\frac{\partial p}{\partial y}\right)_x \left(\frac{\partial v}{\partial x}\right)_y = 0$$

$$\therefore \left(\frac{\partial s}{\partial y}\right)_x - \left(\frac{\partial p}{\partial x}\right)_y = 0 \quad \text{or} \quad \left(\frac{\partial s}{\partial y}\right)_x = \left(\frac{\partial p}{\partial x}\right)_y$$

Replacing  $x$  by  $T$  and  $y$  by  $v$ , we have

$$\left(\frac{\partial s}{\partial v}\right)_T = \left(\frac{\partial p}{\partial T}\right)_v \quad \text{--- (ix)}$$

This is known as Maxwell's second thermodynamical relation.

Multiply both sides of (ix) by  $T$ , we have

$$T \left(\frac{\partial s}{\partial v}\right)_T = T \left(\frac{\partial p}{\partial T}\right)_v$$

$$\text{but } T \partial s = \partial Q$$

$$\therefore \left(\frac{\partial Q}{\partial v}\right)_T = T \left(\frac{\partial p}{\partial T}\right)_v$$