Dr. L. K. Mishra
Deptt of chemistry

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Topic :- Elementary Quantum mechanics.

The schrodinger's wave equation (Continued)

we may now extend the equation (9) for one dimension to three dimensions represented by the Co-ordinates x, y, z.

Evidently f(x) will then be replaced by the amplitude function for the three co-ordinates say $\psi(x, y, z)$. For the sake of simplicity it may be put as ψ

Hence equation (9) will take the form

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} = -\frac{4\pi^2}{\lambda^2}\cdot\psi \quad \text{---(10)}$$

Following de Broglie's idea, schrodinger applied the above treatment to material waves associated with all Particles including electrons, atoms and Photons.

Incorporating the de-Broglie's relationship $\lambda = \frac{h}{mv}$ in equation (10)

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} = -\frac{4\pi^2 m^2 v^2 \psi}{h^2} \quad \text{---(11)}$$

Where m = Mass, V = Velocity of the Particle
The Kinetic energy of the Particle given by $\frac{1}{2} mv^2$ is equal to the total energy E minus Potential energy V of the Particle.

$$K.E = \frac{1}{2} mv^2 = E - V$$

$$mv^2 = 2(E - V) \quad \text{—— (12)}$$

Combining this result with equation (11) we get

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} + \frac{8\pi^2 m(E-V)}{h^2}\psi = 0 \quad \text{(13)}$$

Equation (13) is the well known schrodinger wave equation Proposed by him in 1926.

It is the most Celebrated equation in wave mechanics (or Quantum mechanics) It is customarily written in the following form

$$\left[-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) + V\right]\psi = E\psi \quad \text{(14)}$$

or $$\left[-\frac{\hbar^2}{2m}\nabla^2 + V\right]\psi = E\psi \quad \left(\hbar = \frac{h}{2\pi}\right) \text{—(15)}$$

where ∇^2 (read as del squared) is the laplacian operator defined as

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \quad \text{—— (16)}$$

Defining the Hamiltonian operator $\hat{H}$ as

$$\hat{H} = \frac{-\hbar^2}{2m} \nabla^2 + V \quad \text{—— (17)}$$

Equation (15) becomes $\hat{H}\psi = E\psi$ —(18)

Erwin Schrodinger shared the 1933 Physis Nobel Prize with the British physicist P. A. M. Dirac for formulating the quantum theory of matter.

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