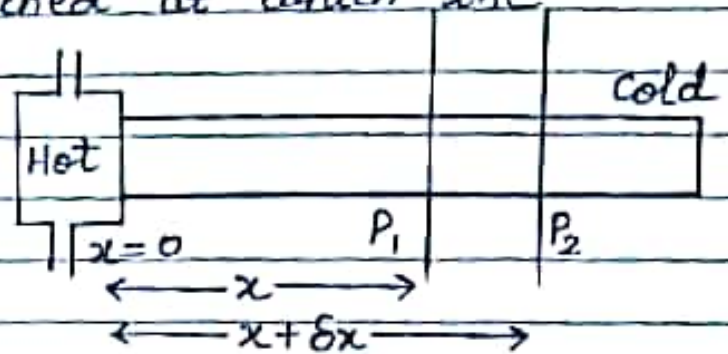


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Fourier's eqn. for heat conduction
Q-1 (11) Paper II

Let us consider a long metal bar of uniform area of cross-section A heated at one end. Heat flows along the length of the bar. Let us suppose that the bar lies along the x -axis of a set of coordinates, whose origin lies at the hot end. In the beginning the temperature at various points along the bar gradually rises but finally a steady state is reached at which the temp. at any point of the bar becomes steady. If the bar is exposed to the surroundings, a portion of the heat passing across any cross section of the bar escapes out from the sides and hence a smaller & smaller amount of heat passes across successive section of the bar.



Let us consider two parallel planes P_1 and P_2 perpendicular to the length of the bar at a distance x and $x + \delta x$ from the hot end. Let θ be the excess of temperature above the surroundings at the plane P_1 and $\frac{d\theta}{dx}$ the temperature gradient, then excess of temperature above the surroundings at

$$P_2 = \left(\theta + \frac{d\theta}{dx} \delta x \right).$$

$$\begin{aligned} \therefore \text{Temp. gradient at } P_2 &= \frac{d}{dx} \left(\theta + \frac{d\theta}{dx} \delta x \right) \\ &= \frac{d\theta}{dx} + \frac{d^2\theta}{dx^2} \delta x. \end{aligned}$$

If θ , Q_1 and Q_2 be the amount of heat that enter the plane P_1 and leave the plane P_2 per second respectively, then

$$Q_1 = -KA \frac{d\theta}{dx}$$

$$\text{and, } Q_2 = -KA \left(\frac{d\theta}{dx} + \frac{d^2\theta}{dx^2} \cdot \delta x \right)$$

Hence, amount of heat gained per second by the element of thickness δx is given by

$$\begin{aligned} Q &= Q_1 - Q_2 = -KA \frac{d\theta}{dx} - \left\{ -KA \left(\frac{d\theta}{dx} + \frac{d^2\theta}{dx^2} \cdot \delta x \right) \right\} \\ &= KA \cdot \frac{d^2\theta}{dx^2} \cdot \delta x \quad \text{--- (3)} \end{aligned}$$

Before the steady state is reached this heat is partly used in raising its temperature. If $\frac{d\theta}{dt}$

is the rate of increase in temp. of the element, ρ the density and S the specific heat of the material, then heat required to raise the temperature of the element at the above stated rate

$$= A \cdot \delta x \cdot \rho S \frac{d\theta}{dt} \quad \text{--- (4)}$$

If P is the perimeter of the element, then surface area of the element $= P \cdot \delta x$.

If E be the radiating power of the surface and θ be the average excess of temp. of the element, the heat radiated per second by the element
 $= E \cdot P \cdot \delta x \cdot \theta$.

Hence,

$$KA \cdot \frac{d^2\theta}{dx^2} \cdot \delta x = A \cdot \delta x \cdot PS \frac{d\theta}{dt} + E \cdot P \delta x \theta$$

$$\text{or, } \frac{d^2\theta}{dx^2} = \frac{PS}{K} \cdot \frac{d\theta}{dt} + \frac{EP}{KA} \cdot \theta \quad \text{--- (5)}$$

This is ^{the} general equation representing the unidirectional rectilinear flow of heat and is called Fourier's differential equation for one dimensional flow of heat.
