D-1 (H)
Paper II

Dr. Supriya Kumari
Deptt. of Physics
J.N.C, Madhubani.

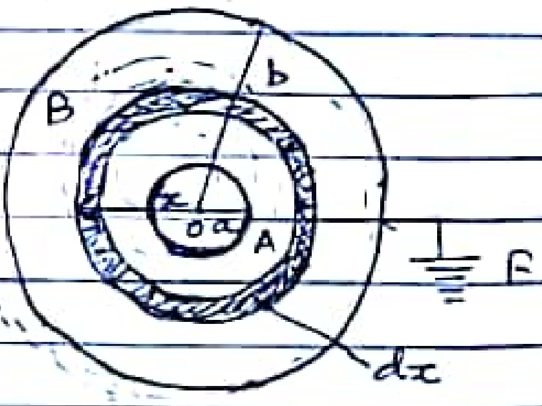
Capacitance of a spherical capacitor

(i) Let A and B be the two concentric spheres of radii a and b separated by air. Let the outer sphere be earthed and a positive charge q be given to the inner sphere.

Now we consider a concentric ~~shell~~ shell of radius x and thickness dx as shown in fig.

Electric field at a point distant x from the centre O, $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2}$

The field acts radially outward.



Fig

Potential difference between the sphere A & B

$$V_A - V_B = \int_b^a -\vec{E}\cdot d\vec{r}$$

$\because$ $\vec{E}$ and $\vec{dr}$ have the same direction say, along the x-axis

$$\therefore \int_b^a -\vec{E}\cdot d\vec{r} = \int_b^a -E\,dx$$

$$= \frac{1}{4\pi\varepsilon_0} \int_b^a -\frac{q}{x^2}\, dx$$

or $$V_A - V_b = \frac{q}{4\pi\varepsilon_0} \left[\frac{1}{x}\right]_b^a$$

$$= \frac{q}{4\pi\varepsilon_0} \left[\frac{1}{a} - \frac{1}{b}\right] = \frac{q}{4\pi\varepsilon_0} \frac{b-a}{ab}$$

$\therefore$ Capacitance of the air capacitor

$$C = \frac{q}{V_A - V_B} = 4\pi\varepsilon_0 \frac{ab}{b-a}$$

(ii) When the space between the two spheres is completely filled with the dielectric of relative permittivity ε_r, the value of E is reduced to E/ε_r as the value charge on the surface of the sphere A is reduced ~~to~~ from q to $\frac{q}{\varepsilon_0}$ due to polarisation of the dielectric.

$$\therefore V_A - V_B = \frac{q}{4\pi\varepsilon_0\varepsilon_r} \frac{b-a}{ab}$$

Hence $$C = \frac{q}{V_A - V_B} = 4\pi\varepsilon_0\varepsilon_r \frac{ab}{b-a}$$