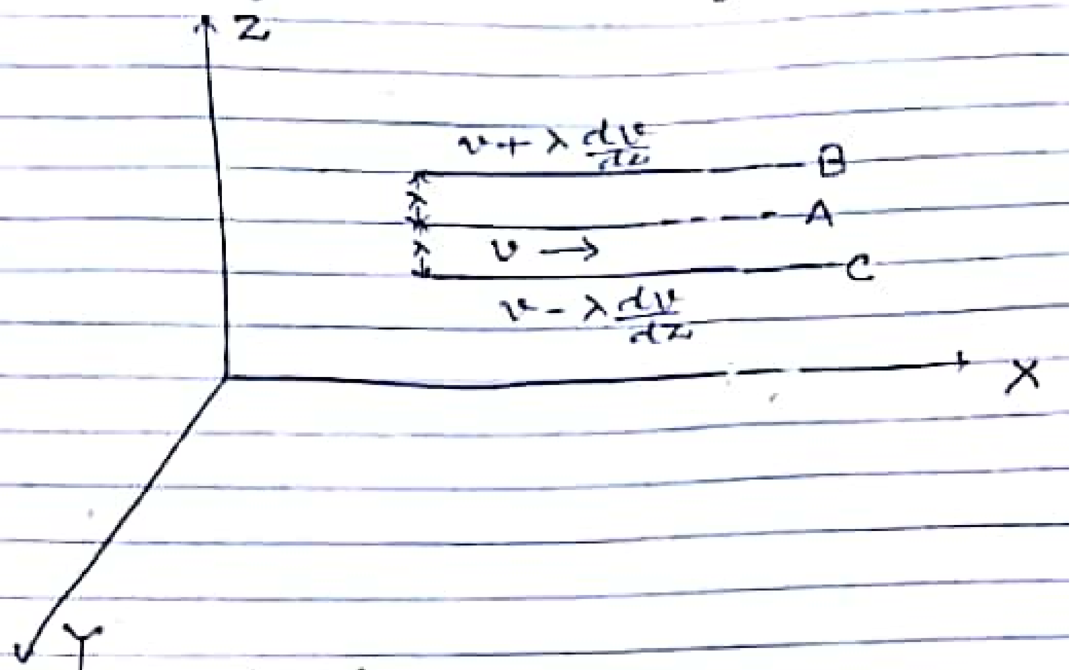
Dr. Supriya Kumari
Deptt of Physics
J N C Madhubani

B-1 (H) Paper II

Transport of momentum
[viscosity of gases]

Viscosity of Gases (transport of momentum)

Let us consider a gas having different velocities from layer to layer. Let us set up an arbitrary co-ordinate system X-Y-Z where we consider three layers A, B, C separated by a distance λ. We also suppose that velocity increases along +z direction



Let the velocity of the layer A $= v$

Then that of layer B $= v + \lambda \frac{dv}{dz}$

and that of layer C $= v + \lambda \frac{dv}{dz}$

where dv/dz = velocity gradient.

If n be number of molecules per unit volume of the gas, then we can say that on the average, $\frac{n}{6}$ molecules per unit volume move along the $+z$ direction.

$\therefore$ No of molecules per unit volume passing through the area dA per unit time on the average

$$= \frac{n\bar{c}\,dA}{6}, \text{ where } \bar{c} = \text{average velocity}.$$

Assuming that the molecules coming from a certain layer carry with them the characteristic momentum of the layer, then the transfer of momentum downwards through the area dA by the molecules from higher velocity layer $= \frac{mnc\,dA}{6}\left(v + \lambda \frac{dv}{dz}\right)$

Similarly, transfer of momentum upwards through the same area dA by the molecules from slower moving layer $= \frac{mn\bar{c}}{6}\,dA\left(v - \lambda \frac{dv}{dz}\right)$

$\therefore$ Net transfer of momentum per second through the area dA

$$= \frac{mn\bar{c}}{6}\,dA\left(v + \lambda\frac{dv}{dz}\right) - \frac{mn\bar{c}}{6}\,dA\left(v - \lambda\frac{dv}{dz}\right)$$

$$= \frac{mn\bar{c}}{6}\,dA\,\lambda\frac{dv}{dz},$$

This gives rise to viscous force, which is equal to $\eta \, dA \frac{dv}{dz}$.

Hence, $\eta \, dA \frac{dv}{dz} = \frac{mn\bar{c}}{3} dA \cdot \lambda \cdot \frac{dv}{dz}$

or $\eta = \frac{1}{3} mn\bar{c}\lambda$ —— (1)

$= \frac{1}{3} \rho \bar{c} \lambda$

Also, $\lambda = \frac{1}{\sqrt{2}\pi\sigma^2 n}$ —— (2)

$\therefore \eta = \frac{1}{3\sqrt{2}} \cdot \frac{m\bar{c}}{\pi\sigma^2}$

Obviously η is independent of n. Since $\bar{c} \propto \sqrt{T}$ hence $\eta \propto \sqrt{T}$. It is also clear that η depends on mass and diameter of the molecules.

Since, at a constant temperature, ρ increases with pressure and λ decreases in the same ratio, η is quite independent of pressure, provided the temperature remains constant.