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Carnot's cycle [contd ---]
D - I (H), Paper II

It is now allowed to undergo a slow adiabatic expansion, performing external work at the expense of internal energy, untill its temperature falls to T_2, the same as that of the sink. The operation is represented by the adiabatic BC. The work done W_2 by the gas is given by

$$W_2 = \int_{V_2}^{V_3} P\,dv = \frac{R(T_1 - T_2)}{\gamma - 1} = \text{area } BbcC \quad \text{--- (2.)}$$

Since the pressure is now very much diminished the gas has lost its expansive power, hence in order to enable it to recover its capacity for doing work it must be brought back to its original condition. To effect this the gas is compressed in two stages: first isothermally along the path CD and then adiabatically along DA. The point D is obtained by drawing the isothermal T_2 through C and the adiabatic through A.

3.) During the isothermal compression, the cylinder is placed in contact with the sink at T_2. The heat which is developed owing to compression will now pass to the sink. This is equal to work done on the gas and is equal to.

$$W_3 = Q_2 = \int_{V_4}^{V_3} P\,dv = RT_2 \log \frac{V_3}{V_4} = \text{area } CcdD \quad \text{--- (3.)}$$

4) The cylinder is again removed to the insulating stand and the gas is compressed adiabatically. The work done on the gas by adiabatic compression is

$$W_4 = \int_{V_1}^{V_4} P\,dv = \frac{R}{\gamma-1}(T_1 - T_2) = \text{area } DdaA \quad \text{—— (4.)}$$

It is thus seen that $W_2 = W_4$.

The net work done by the engine.

$$W = W_1 + W_2 - W_3 - W_4 = \text{Area } ABCD$$

$$= W_1 - W_3 = Q_1 - Q_2 \quad \text{—— (5.)}$$

Efficiency of the Engine: As we have seen, Q_1 is the heat absorbed by the gas during isothermal expansion and Q_2 is the heat rejected by it during isothermal compression, the rest $(Q_1 - Q_2)$ being converted into work.

The efficiency,

$$\eta = \frac{\text{Heat converted into work}}{\text{Heat absorbed from the source}}$$

$$= \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} \quad \text{—— (6.)}$$

Since points B and C lie on the same adiabatic BC, we have

$$T_1 V_2^{\gamma-1} = T_2 V_3^{\gamma-1} \text{ or } \frac{T_1}{T_2} = \left(\frac{V_3}{V_2}\right)^{\gamma-1} \quad \text{—— (7.)}$$

Similarly, since D and A lie on the same adiabatic DA, we have

$$T_2 V_4^{\gamma-1} = T_1 V_1^{\gamma-1} \text{ or } \frac{T_1}{T_2} = \left(\frac{V_4}{V_1}\right)^{\gamma-1} \quad \text{—— (8.)}$$

From (7.) and (8.),

$$\frac{V_3}{V_2} = \frac{V_4}{V_1} \text{ or } \frac{V_2}{V_1} = \frac{V_3}{V_4} \quad \text{—— (9.)}$$

$$\therefore \log \frac{V_2}{V_1} = \log \frac{V_3}{V_4} \quad \text{—— (10.)}$$

$$\therefore \eta = \frac{R\,[T_1 - T_2] \log \frac{V_2}{V_1}}{R T_1 \log \frac{V_2}{V_1}} = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1} \quad \text{—— (11.)}$$

From eq^n (6.) and (11.), we have

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2} \quad \text{—— (12.)}$$

It is thus clear that the efficiency of the engine depends upon temperature T_1 and T_2 of the source and the sink respectively and that the greater the value of $(T_1 - T_2)$, the higher the efficiency. Since, however $(T_1 - T_2)$ must always be less than T_1, it is clear that the efficiency must always be less than 1 or 100 %.