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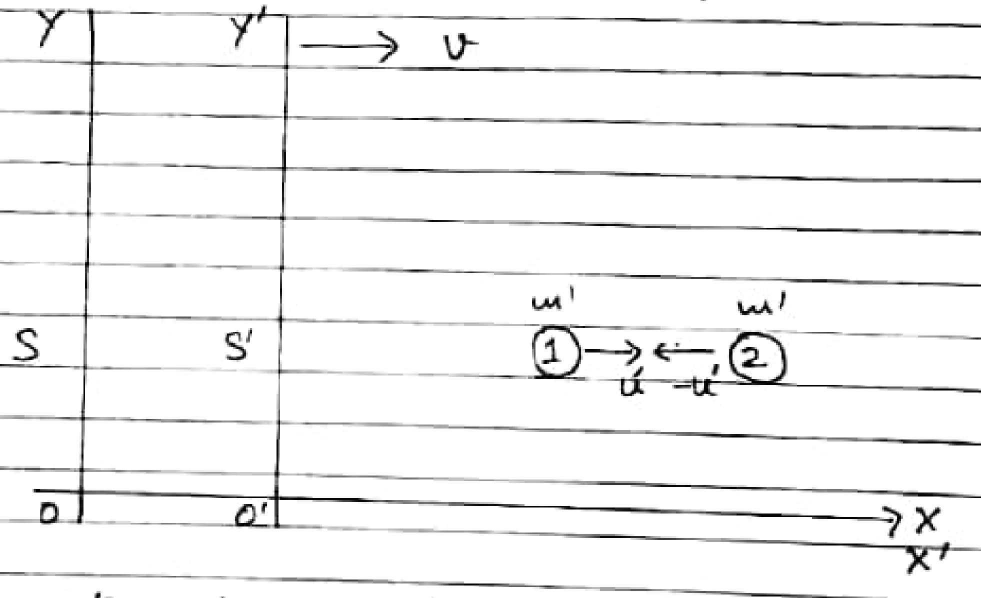
B-1 (H), Paper I

Variation of mass of a particle with its velocity

Ans:- According to the classical Newtonian dynamics, the mass of a moving body is constant independent of velocity. But the theory of relativity leads us to a very different conclusion, viz. the variation of mass with velocity for which an expression can be derived as follows.

Let us consider two bodies of mass m' moving in opposite direction along X-axis with velocities u' & -u' as observed from frame of reference S'. Let these bodies collide and coalesce in to one body. The body thus formed



will be at rest according to the law of conservation of momentum with respect to S'. If the collisions of the two bodies is observed from frame of reference S, the velocities of the two bodies as observed from S will be given by

$$\left.\begin{aligned} u_1 &= \frac{u'+v}{1+\frac{u'v}{c^2}} \\ \text{and} \quad u_2 &= \frac{-u'+v}{1-u'v/c^2} \end{aligned}\right\} \quad \text{— (1)}$$

Let m_1 and m_2 be the masses of the two bodies with respect to frame S. Then the body formed when the two bodies coalesce in to each other has a mass (m_1+m_2) by the law of conservation of mass and it moves with the velocity v along the x-axis with respect to S. It is to be noted that this body is at rest with respect to S'. Then by the law of conservation of momentum we have

$$m_1u_1 + m_2u_2 = (m_1+m_2)v \quad \text{——— (2)}$$

or $$m_1\left(\frac{u'+v}{1+u'v/c^2}\right) + m_2\left(\frac{-u'+v}{1-\frac{u'v}{c^2}}\right) = (m_1+m_2)v \quad \text{——— (3)}$$

Dividing eqn (3) throughout by m_2 and simplying we get

$$\frac{m_1}{m_2} = \frac{1+\frac{u'v}{c^2}}{1-\frac{u'v}{c^2}} \quad \text{——— (4)}$$

Now let us consider the factor $\left(1-\frac{u_1^2}{c^2}\right)$

Substituting the value of u_1 we have

$$1-\frac{u_1^2}{c^2} = 1-\frac{1}{c^2}\left(\frac{u'+v}{1+\frac{u'v}{c^2}}\right)^2$$

$$= \frac{c^2\left(1+\frac{u'v}{c^2}\right)^2-(u'+v)}{c^2\left(1+\frac{u'v^2}{c^2}\right)}$$

$$= \frac{\left(1-\frac{v^2}{c^2}\right)\left(c^2-u'^2\right)}{c^2\left(1+\frac{u'v}{c^2}\right)^2}$$

$$= \frac{\left(1-\frac{v^2}{c^2}\right)c^2\left(1-\frac{u'^2}{c^2}\right)}{c^2\left(1+\frac{u'v}{c^2}\right)^2}$$

$$\therefore\ 1-\frac{u_1^2}{c^2} = \frac{\left(1-\frac{v^2}{c^2}\right)\left(1-\frac{u'^2}{c^2}\right)}{\left(1+\frac{u'v}{c^2}\right)^2} \quad (5)$$

or $$\sqrt{1-\frac{u_1^2}{c^2}} = \frac{\sqrt{\left(1-\frac{v^2}{c^2}\right)\left(1-\frac{u'^2}{c^2}\right)}}{\left(1+\frac{u'v}{c^2}\right)}$$

or $$\left(1+\frac{u'v}{c^2}\right) = \frac{\sqrt{\left(1-\frac{v^2}{c^2}\right)\left(1-\frac{u'^2}{c^2}\right)}}{\sqrt{1-\frac{u_1^2}{c^2}}} \quad (6)$$

Similarly $$\left(1-\frac{u'v}{c^2}\right) = \frac{\sqrt{\left(1-\frac{v^2}{c^2}\right)\left(1-\frac{u'^2}{c^2}\right)}}{\sqrt{1-\frac{u_2^2}{c^2}}} \quad (7)$$

Substituting these values in equn (4) we have

$$\frac{m_1}{m_2} = \sqrt{1-\frac{u_2^2}{c^2}} \Big/ \sqrt{1-\frac{u_1^2}{c^2}} \quad (8)$$

If the velocity of the second body as observed with respect to S is zero, i.e $u_2 = 0$ then its mass m_2 can be denoted by m_0. The symbol m_0 gives the mass of a body when it is at rest with respect to the frame of reference being used. Let $u_1 = v$ i.e velocity of the first body with respect to S is v. we can write $m_1 = m$ then equn (8) becomes

$$\frac{m}{m_0} = \frac{1}{\sqrt{1-v^2/c^2}}$$

or $$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \quad (9)$$

This is the relativistic formula for the variation of mass with velocity. m_0 is called the rest mass that is when the body whoe mass is measured is at rest relative to the observer, m is the effective mass that is the mass of the body when it is moving with the velocity v with respect to the observer. Hence as far as the observer is concerned masses of any moving system appear to increase with velocity, becoming infinite when v attains the velocity of light c. This indicates that c is a limiting velocity unattainable by moving material body. when v is small compared with c, $\frac{v^2}{c^2}$ becomes negligible and the mass of the body remains sensibly constant. This law has been directly varified by the experiments of Kaufmann, Bucherer and Guye & Lavanchy on high speed electrons. Many other observations also have confirmed its validity.