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Lorentz Transformation equations.
D-1 (H) Paper I.

Q:- State the postulates of the special theory of relativity and deduce from them the Lorentz transformation equations.

Ans:- The postulates of spl. relativity theory :-

(i) The laws of physics are the same in all inertial systems. No preffered inertial system exists.

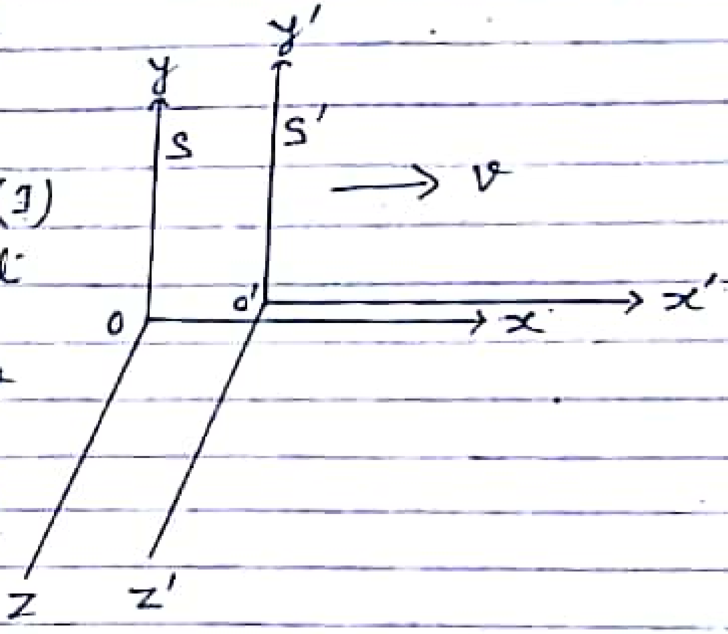
(ii) The speed of light in free space has the same value c in all inertial systems.

Lorentz transformation eqs. :- The eqn. in relativity physics which relate to the space and time co-ordinates of two co-ordinate systems moving with a uniform velocity relative to one-another are called Lorentz transformation.

Let us consider two observers O & O' located in two separate inertial co-ordinate system S & S' respectively. The system S' moves with a uniform velocity v to the right along the x-axis relative to S. This is equivalent to the motion of S to the left with a velocity v relative to S'.

Any event in S must have a single & unique interpretation in the system S. Therefore, the two transformation equations in space co-ordinates and in time must be linear. It is easy and resonable to guess that the single simple linear equations between

x & x' must be of the form



$$x' = k(x - vt) \quad \text{—— (1)}$$

where k is a constant of proportionality that is independent of x & v but t may depend upon v. The corresponding eqn. for x in terms of x' & t' can be written as

$$x = k(x' + vt') \quad \text{—— (2)}$$

There is obviously no difference between the corresponding co-ordinates y, y' and z, z' as they are perpendicular to the direction of v. So, we have

$$y' = y \quad \text{—— (3)}$$

$$z' = z \quad \text{—— (4)}$$

The time co-ordinates t and t' are however not the same. Substituting the value of x' from eqn. (1) in eqn (2) we have

$$\begin{aligned} x &= k[k(x - vt) + vt'] \\ &= k^2(x - vt) + kvt' \\ &= k^2x - k^2vt + kvt' \end{aligned}$$

$$kvt' = x(1-k^2) + k^2vt$$

$$t' = kt + \left(\frac{1-k^2}{kv}\right)x \quad \text{---} (5)$$

Eqn. (1), (3), (4) & (5) constitute a co-ordinate-transformation in space & time which satisfies the first postulate of relativity.

The value of k can be evaluated from second postulate of relativity.

Let us imagine that at the time $t = t' = 0$ when the origin O' coinsides with the origin O a spherical pusle of light leaves the common origin of S and S'. As the velocity of light is invarient each observer sees a spherical wave expanding outwards with the speed c in his own system as measured by his own meter stick & clock. For the observer O the distance travelled by light in a certain time along x axis is given by

$$x = ct \quad \text{---} (6)$$

and for O' it is given by

$$x' = ct' \quad \text{---} (7)$$

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