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Lorentz Transformation equations.

D-1(H) Paper I.

[Continued ---]

(7)

Substituting the value of x' and t' from eqn (1) & (5) in eqn (7) we have

$$k(x - vt) = c \left[kt + \left(\frac{1 - k^2}{kv} \right) x \right]$$

$$\text{or } kx - \left(\frac{1-k^2}{kv}\right)cx = ckt + kv t$$

$$\text{or } x \left\{ k - \left(\frac{1-k^2}{kv}\right)c \right\} = ckt + kv t$$

$$\text{or } x = \frac{ckt + kv t}{k - \frac{(1-k^2)c}{kv}}$$

$$= \frac{ckt \left(1 + \frac{v}{c}\right)}{k \left[1 - \frac{(1-k^2)c}{k^2 v}\right]}$$

$$= \frac{ct \left(1 + \frac{v}{c}\right)}{1 - \left(\frac{1}{k^2} - 1\right)\frac{c}{v}}$$

$$x = \frac{x \left(1 + \frac{v}{c}\right)}{1 - \left(\frac{1}{k^2} - 1\right)\frac{c}{v}}$$

$$\therefore 1 + \frac{v}{c} = 1 - \frac{c}{v} \cdot \frac{1}{k^2} + \frac{c}{v}$$

$$\frac{c}{v} \cdot \frac{1}{k^2} = \frac{c}{v} - \frac{v}{c} = \frac{c^2 - v^2}{vc}$$

$$\frac{1}{k^2} = \frac{c^2 - v^2}{c^2} = \frac{1 - \frac{v^2}{c^2}}{1}$$

$$\frac{1}{k} = \sqrt{1 - \frac{v^2}{c^2}}$$

$$\text{or } k = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (8)$$

$$x = k [k(x - vt) + vt']$$

$$= k^2 (x - vt) + kv t'$$

$$= k^2 x - k^2 vt + kv t'$$

Substituting the values of k in eqns (1) & (5) we have

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}$$

$$y' = y$$

$$z' = z$$

$$\text{and } t' = \frac{t - vx/c^2}{\sqrt{1 - v^2/c^2}}$$

These are Lorentz transformation eqns. In order to transform measurements from S' to S we have to replace v by $-v$. Thus the inverse Lorentz transformation eqns are

$$x = \frac{x' + vt'}{\sqrt{1 - v^2/c^2}}$$

$$y = y'$$

$$z = z'$$

$$t = \frac{t' + vx'/c^2}{\sqrt{1 - v^2/c^2}}$$

Two significant aspects of Lorentz transformation eqns are

- (1) The measurement of time as well as position depends

depends upon the frame of reference of the observer so that two events which occur simultaneously in one frame of reference need not be simultaneous when viewed from another.

(2.) If the relative velocity v of the frame S' relative to S is very small as compared to the velocity of light c , the Lorentz relativistic transformation reduced to the ordinary Galilean eqⁿ. i.e

$$x' = x - vt$$

$$y' = y$$

$$z' = z$$

$$\text{and } t' = t.$$