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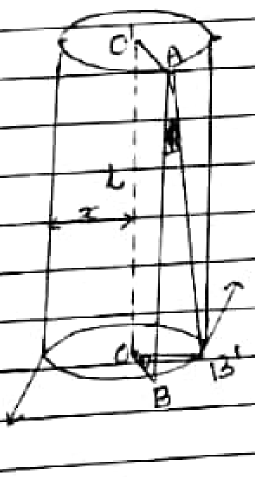
Twisting Couple on a Cylinder

D - 1 (H) Paper I

Let us consider a cylinder of length l and radius r, of a material of coefficient of rigidity n be fixed at upper end and a couple be applied to the lower end in a plane perpendicular to its length twisting it through an angle θ (radians). This is an example of 'Pure' Shear; for, the twist produces a change neither in length or radius of the cylinder.

Now, in the position of equilibrium, the twisting couple is equal and opposite to the restoring couple.

Let us imagine the cylinder to consist of a large number of co-axial, hollow cylinders, and consider one such hollow cylinder of radius x, and radial thickness dx.



Let AB be a line on its surface initially perpendicular to its fixed ends. After the cylinder is twisted let B move to B'. A will not change its position as it is a point on clamped end. Then $\angle BOB' = \theta$ is called the angle of twist. Hence angle of twist is the angle through which the radius of the free end is turned. Angle $BAB' = \phi$ is the shearing strain at the surface of the hollow cylinder, whereas θ is the same for all the hollow cylinders, φ changes from cylinder to cylinder and it is maximum at the outermost surface and zero at the axes OO'. From the fig

$$BB' = l\phi \quad \& \text{ also } BB' = x\theta$$

$$\therefore l\phi = x\theta$$

$$\therefore \phi = \frac{x\theta}{l}$$

If F be the shearing stress on the surface of the cylinder, then rigidity

$$\eta = \frac{F}{\phi} = \frac{Fl}{x\theta}$$

$\therefore F = \frac{n x \theta}{l}$

Now the face area of the hollow cylinder

$= 2\pi x \, dx$

$\therefore$ Total shearing force on this area

$= 2\pi x \, dx \times n \frac{x\theta}{l}$

$= 2\pi n \frac{\theta}{l} x^2 dx$

Therefore, moment of this force about the axes OO' is equal to $= 2\pi n \frac{\theta}{l} x^2 dx \times x$

$= \frac{2\pi n \theta x^3 dx}{l}$

Integrating this expression between the limits $x = 0$ to $x = r$, we have

Total twisting couple on the cylinder

$= \int_0^r \frac{2\pi n \theta x^3 dx}{l}$

$= \frac{2\pi n \theta}{l} \int_0^r x^3 dx$

$= \frac{2\pi n \theta}{l} \left[\frac{x^4}{4}\right]_0^r$

$= \frac{2\pi n \theta r^4}{4l}$

$= \frac{\pi n \theta r^4}{2l}$

If $\theta = 1$ radian, we have twisting couple per unit twist of the cylinder or wire

$C = \frac{\pi n r^4}{2l}$

This twisting couple, per unit twist of the wire is also ~~as~~ called the torsional Rigidity.
