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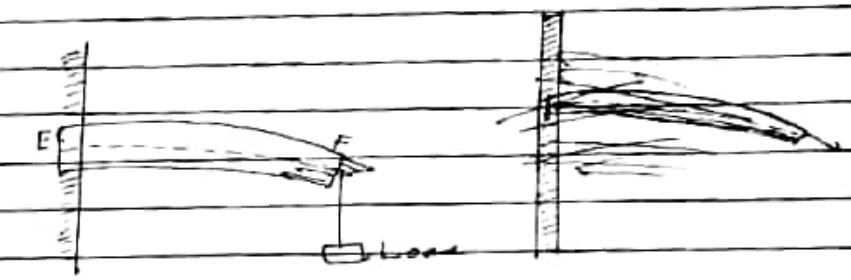
Bending of beams

B-I(H), Paper III

A beam is a rod or a bar of uniform cross-section (circular or rectangular) of a homogenous ~~Cross-section~~, isotropic elastic material, whose length is very great compared with its thickness, so that the shearing stresses over any section are negligibly small and the bending is pure or simple.

When such a beam is fixed at one end and loaded at the other, it is called a Cantilever. In this case the loaded end sinks a little. As a result the beam gets bent under the action of the couple, due to the load applied as shown in fig. The upper surface of the beam thus gets stretched and assumes a convex form and its lower surface gets compressed and assumes a concave form.



The extensions and contractions are however not confined only to the upper and the lower surfaces of the beam respectively but also take place inside the body of the beam, all longitudinal filaments in the upper half getting extended and, in the lower half, getting compressed. This is shown in fig by means of arrows, whose lengths indicate in rough proportion the extent of the extensions and contractions of the filaments in the upper & lower halves of the beam respectively.

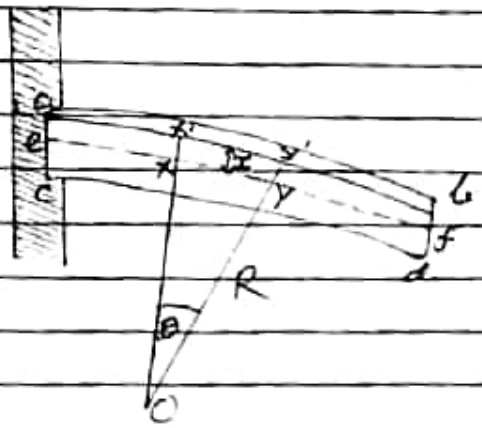
The extension is the maximum in the uppermost filament and the compression, the maximum in the lowermost ones, both decreasing progressively as we proceed towards the axis of the beam from either side.

So that, in the plane perpendicular to the section of the beam containing the axis EF, the filaments neither get extended nor shortened. This surface is called the neutral surface and its section by the plane of bending ~~or ABCD~~ which is perpendicular to it called neutral axis

Bending Moment: When the rod is fixed at one end and loaded at the other a bending is produced due to the moment of the load. The deformation produced by the load brings about restoring forces due to elasticity tending to bring the rod back to its original position. In equilibrium

Restoring Couple = Bending Couple

These two couples act in the opposite directions



Let us consider a small part XY of the neutral axis of the rod ~~strip~~ bent into an arc of radius R subtending an angle θ at the centre of curvature O as shown in fig. ~~Let x'y'~~

Let x'y' be another filament at a distance x from the neutral surface, then

$$\theta = XY/R$$

$$\text{or } XY = R\theta$$

$$\text{and } x'y' = (R+x)\theta$$

Increase in length $= x'y' - XY$

$$= (R+x)\theta - R\theta$$

$$= x\theta$$

$$\therefore \text{Strain} = \frac{\text{Change in length}}{\text{Original length}} = \frac{x\theta}{R\theta} = \frac{x}{R}$$