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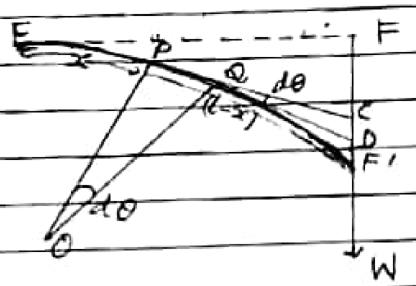
Cantilever :- Q-1(s)

Cantilever. It is a beam fixed horizontally at one end and loaded at the other.

Let EF represent the neutral axis of the Cantilever of length l fixed at the end E and loaded with a load W at F. as shown in fig. Let the end F be deflected to the position F′ under the action of the load.



Let us consider a section of the beam as at P at a distance x from the end E. Then the moment of the Couple due to the load W

$$= W(l-x)$$

As the point F′ is very close to F, PF′ is almost perpendicular to FW.

Since the beam is in equilibrium, the bending couple is equal to the restoring couple

YI_g/R, where R = Radius of curvature of the neutral axis at P
Y = Young's mod
I_g = Geometrical M.I.

$$\text{Hence } W(l-x) = \frac{YI_g}{R}$$

$$\text{or } \frac{1}{R} = \frac{W(l-x)}{YI_g}$$

As we move towards the fixed end E, the moment of the load increases. Hence the radius of curvature will be different at different points and decreases as we move towards it. Let Q be another point very close to P at a distance dx.

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As the dx is very small, the radius of curvature at Q is practically the same

$$\therefore \quad d\theta = \frac{dx}{R} = \frac{W(l-x)\,dx}{Y I_g}$$

Let us draw tangents at P and Q meeting the vertical line at C & D respectively. Then the depression of Q below P is evidently

$$CD = dy = (l-x)\,d\theta$$

where $d\theta$ = Angle between two tangents.

$$\therefore \quad dy = (l-x)\,\frac{W(l-x)\,dx}{Y I_g}$$

$$= \frac{W}{Y I_g}(l-x)^2\,dx$$

$\therefore$ Total depression $FF' = y = \int_0^l \frac{W}{Y I_g}(l-x)^2\,dx$

$$= \frac{W}{Y I_g}\int_0^l (l^2 + x^2 - 2lx)\,dx$$

$$= \frac{W}{Y I_g}\left[l^2 x + \frac{x^3}{3} - \frac{2 l x^2}{2}\right]_0^l$$

$$= \frac{W}{Y I_g}\left[l^3 + \frac{l^3}{3} - l^3\right]$$

$$= \frac{W l^3}{Y I_g}$$

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