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Gradient of a Scalar field
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Gradient of a Scalar field: Let us consider a point P in the Scalar field having position vector $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and let the value of the scalar field at this point be $\phi(\vec{r}) = \phi(x,y,z)$ where $\phi(x,y,z)$ is continuous differentiable function of three independent space Co-ordinates x, y, z

The partial derivative of ϕ with respect to x, keeping y and z Const $= \frac{\partial \phi}{\partial x}$.
It measures the rate of change of ϕ at the point P along the x-direction
Similarly the rate of change of ϕ at P along the Y-direction $= \frac{\partial \phi}{\partial y}$

and the rate of change of ϕ at P along the z-direction $= \frac{\partial \phi}{\partial z}$

The function, ϕ, Therefore has different rates of variation along different directions. With the partial derivatives $\frac{\partial \phi}{\partial x}, \frac{\partial \phi}{\partial y}, \frac{\partial \phi}{\partial z}$ we can ~~Graph~~ Construct at every point in space a vector whose Components along the x, y and z directions are equal to the respective partial derivatives of the Scalar function ϕ. This vector is known as gradient of ϕ or grad ϕ

$$\therefore \text{grad}\,\phi = \frac{\partial \phi}{\partial x}\hat{i} + \frac{\partial \phi}{\partial y}\hat{j} + \frac{\partial \phi}{\partial z}\hat{k} \quad \text{---(i)}$$

$$= \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\phi$$

$$= \vec{\nabla}\phi \quad \text{---(ii)}$$

The Symbol $\vec{\nabla}$ is called vector differential operator or Del operator. It should be clearly understood that $\vec{\nabla}$ is not a vector but an operator which obeys laws of vectors. From relation (i) we have

$$\text{Grad}\,\phi = \vec{\nabla}\phi = \frac{\partial \phi}{\partial x}\hat{i} + \frac{\partial \phi}{\partial y}\hat{j} + \frac{\partial \phi}{\partial z}\hat{k}$$

This shows that grad ϕ is a vector whose x, y, z Components are $\frac{\partial \phi}{\partial x}\hat{i}$, $\frac{\partial \phi}{\partial y}\hat{j}$ and $\frac{\partial \phi}{\partial z}\hat{k}$
Hence the gradient of a scalar field is a vector

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For example, electric potential V is a scalar function but $\vec{\nabla} V = \vec{E}$, the electric field intensity is a vector which shows how V varies in the neighbourhood of a point