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Divergence of a vector field

Divergence of a vector field: Just as we can get a vector field starting from a scalar field, we can get a scalar field starting from a vector field.

Let us consider a vector field $\vec{A}$, the magnitude and direction of which is a function of position co-ordinates at a point, then the scalar product of differential operator $\vec{\nabla}$ and the vector $\vec{A}$ is a scalar function of position co-ordinates x, y, z and is known as the divergence of vector $\vec{A}$

$$\therefore \quad \text{div}\,\vec{A} = \vec{\nabla}\cdot\vec{A}$$

where $\vec{\nabla} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)$

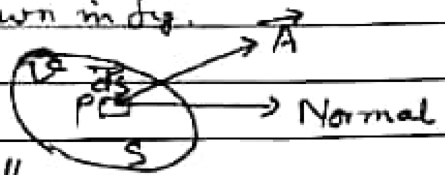
and $\vec{A} = \left(\hat{i}A_x + \hat{j}A_y + \hat{k}A_z\right)$

$$\therefore \quad \vec{\nabla}\cdot\vec{A} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\cdot\left(\hat{i}A_x + \hat{j}A_y + \hat{k}A_z\right)$$

$$= \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

Physical Meaning: The divergence of a vector is the limiting value of the net outward flow of some physical quantity like a fluid or electric flux through the surface area of unit volume as the volume tends to approach zero.

Let us consider a closed volume v having a surface area S in a vector field as shown in fig.



Let the vector field at a point P on the surface be $\vec{A}$ and $\vec{ds}$ be a vector representing small area surrounding the point P, its direction being that of the outward drawn normal to the surface

If θ be the angle between the vector and the outward drawn normal to the surface at P, Then
Flux of the field out of the area $\vec{ds}$
$$= A\, ds \cos\theta = \vec{A} \cdot \vec{ds}$$
$\therefore$ Total flux out of the whole surface
$$= \iint_S \vec{A} \cdot \vec{ds}$$

If the volume enclosed by the surface be vanishingly small then the outward flux per unit volume gives the divergence of $\vec{A}$ or $\operatorname{div}\vec{A}$

$$\therefore \quad \operatorname{div}\vec{A} = \lim_{v \to 0} \frac{1}{v} \iint_S \vec{A} \cdot \vec{ds}$$